Variety provision of a multiproduct monopolist

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Abstract

We investigate a multiproduct monopolist's provision of variety and identify conditions of consumer preferences (demand structure) under which the monopolist over-provides or under-provides variety compared to the second-best, that is, the total welfare maximizing variety constrained by the firm's price/quantity choice. We then illustrate how the previous conditions of under-/over-provision of variety differ if consumers face intrinsic (search) costs to learn their tastes for products. We discuss important applications of this analysis, such as variety provision of retailers and variety provision on e-commerce platforms, like eBay. We also link variety provision to quality provision and illustrate how the monopolist's quality provision compares to the second-best optimal quality, which, for instance, can be used to set minimum quality standards.

Keywords: Multiproduct monopoly, variety provision, taste distributions
JEL Codes: D42, L12, L15

1 Introduction

Most firms sell multiple products; retail stores sell products of competing brands in each category, e-commerce platforms, like eBay, Amazon, sell products of different sellers. Number of differentiated products (variety) that a store or a platform offers to

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consumers determines consumers' choice set for a given shopping trip. Optimal variety provision of a multiproduct firm and how it compares to the socially optimal level are fundamental questions in economics, yet there is very limited work on this (see below). Besides, optimal variety provision in presence of transportation/search costs has a particular importance for retailing and e-commerce. Due to search costs many consumers might choose to be one-stop shoppers (or single-homers), that is, they visit one retail store (or one platform), and this is more likely to be the case if they aim to buy one product at a given time. Thus, variety provision of a multiproduct firm might determine choice sets of many consumers and significantly affect what consumers buy. It is therefore important to know main determinants of variety offered by a multiproduct firm and how this compares with the social welfare-maximizing variety level in the presence of search costs. These fundamental questions are particularly important in the context of e-commerce given increasing significance of e-commerce platforms for retail trade.[1]

This paper studies a multiproduct monopolist's provision of variety and identifies conditions of consumer preferences (demand structure) under which the monopolist over-provides or under-provides variety compared to the "second-best", that is, the total welfare maximizing variety constrained by the firm's price/quantity choice. We then illustrate how the previous conditions of under-/over-provision of variety differ if consumers face intrinsic (search) costs to learn their tastes for products. We discuss important applications of the analysis with search costs, such as variety provision of retailers and variety provision on e-commerce platforms, like eBay and Amazon. Besides, we provide a framework capturing important characteristics of online market places and can thus be used to analyze e-commerce platforms' optimal seller fees and the implied variety offered to consumers. We also link variety provision to quality provision and illustrate how the monopolist's quality provision compares to the second-best optimal quality, which, for instance, can be used to determine minimum quality standards.

In the benchmark model of the multiproduct monopolist selling symmetrically differentiated products, we firstly consider the case where consumers have no search costs. We compare the monopolist's variety (total number of products) to the social

[1] In 2017 Business-to-Consumers (B2C) e-commerce turnover was 514 billion euros (5 percent of GDP) in the EU, making 8.8 percent of its retail trade. In 2017, B2C e-commerce accounts for 9 percent of total retail trade in the US and 23.8 percent of total retail trade in China, see Global E-commerce Report (2017).

planner's variety at a given total quantity (similar to Spence [1975] analysis of quality provision by the monopolist) and thereby link the monopolist's variety provision to the monopolist's quality provision. We also compare the monopolist's variety with the second-best optimum, where the social planner is constrained by the monopolist's choice of total quantity (or price). We also identify conditions under which a particular form of Spence distortion implies a particular form of distortion with respect to the second-best.

We have three key takeaways from the results of the benchmark analysis: 1) Consumer preferences for variety (how tastes for different products are distributed) are crucial to determine whether the multiproduct monopolist distorts variety and which way this distortion goes, 2) When demand for products is given by commonly used Multinominal Logit (MNL), the monopolist provides right amount of variety at a given total quantity (no Spence distortion), but under-provides variety compared to the second-best. This result is particularly important for empirical work that uses MNL demand to analyze optimal variety provision of a multiproduct firm. Due to the structure of MNL demand, the monopolist always under-provides variety, whereas this result might be different for other demand specifications as we show next, 3) For a more general distribution of tastes in discrete choice demand model, whether and which way the monopolist distorts variety depends on log-log concavity properties of the taste distribution function, F. For instance, if the taste distribution is Extreme Value Type I ($-logF$ is log-linear), we show that there is no Spence distortion and the monopolist under-provides variety compared to the second-best. If the taste distribution function is exponential ($-logF$ is strictly log-convex), we show that the monopolist under-provides variety in Spence terms and also compared to the second-best. If $-lnF$ is strictly log-concave, we show that the monopolist over-provides variety in Spence terms, which might be the case for uniformly distributed tastes. For instance, when products are horizontally differentiated on the Salop circle, the monopolist over-provides variety compared to the second-best optimal variety (and also compared to the first-best optimal variety).

In many settings consumers can learn about their tastes only after incurring some costs. In brick-and-mortar retail markets consumers need to visit stores and inspect products to discover what they like and which product they prefer. In e-commerce consumers need to spend time online by comparing offers of different sellers for the product they are interested in. To capture such widely observed situations we next extend the benchmark model by assuming that consumers incur intrinsic search costs in order to learn their tastes for products. Such a change in our model leads to two elastic margins: 1) Extensive margin where consumers decide whether to visit the shop and incur costs to discover their tastes for products, 2) Intensive margin where consumers visiting the shop decide whether to buy a product and if so, which one to buy. Our main focus is to investigate how the monopolist's provision of variety is compared to the optimal variety level(s) when the multiproduct monopolist's variety and price choices affect the extensive margin and the intensive margin in a distinct way. The key question is how this analysis compares to the standard multiproduct firm setting (our benchmark), where consumers' costs of visiting the shop are so low that all consumers visit the shop (there is only elastic transaction demand). The answer to this question will enable us to see how the existence of binding search costs at participation stage are important for variety provision by multiproduct firms, like retailers, e-commerce platforms.

There are three key takeaways from the analysis with search costs (two margins): 1) When per-consumer demand is MNL, we have the same results as the benchmark without search costs: the monopolist provides right amount of variety at a given total quantity (no Spence distortion) and the monopolist under-provides variety compared to the second-best (optimal variety constrained by the monopolist's price/quantity choice), 2) Log-log concavity properties of taste distribution F determine whether intensive margin is relatively more elastic to variety changes than price changes compared to the extensive margin, 3) Spence distortions identified when there was only one elastic margin (without search costs) have opposite directions when there is elastic participation and transaction margin. More precisely, we show that when $-logF$ is log-convex, the intensive margin is relatively more elastic to variety changes than price changes compared to the extensive margin. In that case, the monopolist over-provides variety at a given total quantity, whereas there was under-provision of variety in Spence terms when consumers faced no search costs and $-logF$ is strictly log-convex. Symmetrically, when $-logF$ is sufficiently log-concave, the intensive margin is relatively less elastic to variety changes than price changes compared to the extensive margin. In that case, the monopolist under-provides variety at a given total quantity, whereas there was over-provision of variety in Spence terms when consumers faced no search costs and $-logF$ is strictly log-concave.

These results illustrate that introducing search frictions changes variety provision incentives of the monopolist dramatically and makes over-provision of variety more likely. Intuitively, when consumers incur search costs to learn their tastes for products, there is ex-ante uncertainty about how much consumers benefit from visiting the shop, since once in the shop consumers might choose not to purchase any product from the monopolist if the value of outside option is higher than their surplus from the best match. This ex-ante uncertainty shifts total consumer demand downward. In order to convince consumers to visit the store, the monopolist might want to offer a larger portfolio of products which increases the consumer expected surplus from being matched to their best product and so increases consumer participation. How more variety affects consumer participation demand (extensive margin) depends on how much the extensive margin changes with variety relative to how much it changes with price vs how much intensive margin changes with variety relative to how much it changes with price. This is because when the firm offers more variety, the direct effect of variety increases consumers' expected surplus from visiting the store, whereas the indirect effect of variety (via higher prices) lowers their expected surplus. Which effect dominates depends on the relative elasticities of extensive and intensive margins.

For instance, suppose the intensive margin is relatively more elastic to variety than price compared to the extensive margin ($-InF(z)$ is log-convex). If the firm offers more variety, its total demand goes up both due to more people visiting the store and due to more of these visitors purchasing a product from the firm. Suppose that the firm raises its prices to keep the total demand constant. Such a change implies that at new prices less consumers visit the firm, that is, the expected surplus from visiting the store decreases (extensive margin decreases) (given intensive margin is relatively more elastic to variety than price compared to the extensive margin). Thus, in this case the firm over-provides variety at a given total quantity level. Symmetric argument applies when when the intensive margin is relatively less elastic to variety than price compared to the extensive margin ($-InF(z)$ is sufficiently log-concave). In this case the firm can attract more consumers, that is, the expected surplus from visiting the store increases (extensive margin increases) by increasing its variety and prices while keeping the total demand constant. This is the case where the firm under-provides variety at a given total quantity level.

In trade platform application, we firstly develop a framework of one platform capturing important facts of e-commerce: The platform sets a listing (fixed) fee and transaction fee for sellers. Sellers then decide whether to post their product by paying the listing fee to the platform. Each seller that posts its product on the platform sets its price to consumers and for each purchase on the platform the seller of the product collects its price and pays a (transaction) fee to the platform. On the other hand, buyers do not pay any fee to the platform, but they incur an intrinsic (search) cost to enter the platform. Before entering the platform (ex-ante) buyers do not know their tastes (match value) to the products, which they discover once they incur the search cost and enter the platform. We assume free entry of sellers to the platform and so the number of sellers (available variety) on the platform is endogenously determined. In this setup we first show that the platform's problem of setting a listing fee and a transaction fee is mathematically equivalent to the multiproduct monopolist's problem of setting its variety and prices when consumers incur search costs to discover their tastes for products. Intuitively, the platform captures sellers' surplus via a fixed fee and so internalizes the entire profits from trade. The equivalence result implies that the platform can coordinate independent sellers' pricing via its choice of seller transaction fees and thereby eliminate competition between sellers, and determines the number of sellers (variety) on the platform via its choice of seller listing fee. The equivalence result also enables us to understand the platform's optimal variety and price choices, and how they compare to the socially optimal levels using the results from the multiproduct monopolist's analysis. It is important to note that the equivalence result holds both when sellers are symmetrically differentiated and when they are asymmetric in quality and the platform can perfectly price discriminate (set a different seller fee contract to each type of seller). We also show that the equivalence holds if we allow the platform to charge an ad-valorem fee (instead of a constant unit fee) to sellers in addition to a fixed fee.

We extend the trade platform setup to the case of asymmetric sellers (products) in quality. We show that when demand for products are given by MNL, the platform wants to implement the same markup for all products. Asymmetric sellers in equilibrium will set different markups: higher quality sellers will set higher markups. In order to achieve the same markup for all products, the platform sets lower unit commission on higher quality products. And the equilibrium fixed fee will be higher for higher quality products. Next, we consider selection of products into the platform. To do that we investigate how the platformÕs equilibrium seller fees for one product change if the platform replaces this product with a higher quality alternative. Our preliminary insight is that the platform sets higher seller fees to an entrant seller if the entrant wants to replace a lower quality product listed on the platform. This therefore generates inefficient entry costs for higher quality sellers who would like to list their product in the trade platform. This ineffiency arises because when the platform sells a higher quality product, its total demand increases more than the demand for the replaced product. In other words, inefficiently high commissions to a better-quality entrant seller is due to the platform behaving like a multiproduct monopolist and setting the same markup for all products. We currently investigate welfare properties of the platform's variety provision in the context of heterogenous sellers in quality. We also want to study how unobserved seller heterogeneity would affect the equivalence result, welfare properties of the platform's variety provision, and the comparison of unit seller fee contracts with percentage commissions.

Our results might potentially have important policy implications for variety provision in e-commerce platforms. The European Commission fined Google for 2.42 billion euros in 2017 for distorting its search algorithm (COMP/39740). After the restrictions on the use of Most-Favored-Customer clauses (MFCs) are implemented in Europe (see, for instance, German anti-trust authority's cases, B9-66/10, B9-121/13), competition authorities are worried that price comparison websites distort their search algorithm to disfavour sellers who offer their products cheaper at a different outlet. Our results suggest that such search distortions in online markets would affect not only prices but also variety provision by e-commerce platforms. Furthermore, identifying conditions under which percentage seller commissions are better/worse than constant unit fee seller contracts will enable policy makers to see in which product categories percentage commissions should be allowed/banned or how to design online sales taxes to undo such distortions.

1.1 Related Literature

A classical question in welfare economics is whether the market provides optimal variety. A general conclusion is that in many market specifications over-provision of variety prevails [Chamberlin Edward, 1933, Mankiw and Whinston, 1986, Salop, 1979, Anderson et al., 1995], whereas under-provision can also arise in some cases [Dixit and Stiglitz, 1977, Spence, 1975, Lancaster, 1979]. This literature mostly covers single-product firms with free entry conditions. This paper contributes to the existing literature by providing an analysis of optimal variety provision of a multiproduct monopolist where consumers' benefits from variety increase at a decreasing rate and the firm incurs a constant unit cost for an additional variety. We analyze this in two cases: when consumers incur no search costs and when consumers incur search costs to learn their tastes for products. In the latter model of mutiproduct monopolist consumers make two distinct decisions (visiting the store and purchasing something) and these decisions depend on pricing and the number of variants offered by the firm.

Spence [1975] analyzes the equilibrium provision of quality by a monopolist and shows that the monopolist under-provides quality at a given quantity if the average consumer's valuation for quality is greater than that of the marginal consumer. "Quality" can be interpreted as "variety" since they both shift the demand curve upwards and imply additional costs to the firm. Our multiproduct monopoly analysis builds on Spence [1975]. Going beyond Spence [1975] analysis, we illustrate under which conditions the monopolist under-/over-provides quality with respect to the second-best optimum. Besides, in the multiproduct firm framework we illustrate conditions on consumer preferences (demand systems) for differentiated products that imply under-/over-provision of variety in Spence terms (at a given total quantity) as well as compared to the second-best optimum. Furthermore, we contribute to this literature by illustrating how having both extensive and intensive margins matters for the monopolist's provision of variety compared with the social optimality benchmarks.

There is very limited research available on provision of variety in markets with platforms [Nocke et al., 2007, Hagiu, 2009]. These papers differ significantly from our trade platform setup: 1) they consider a model of only membership decisions (all buyers and sellers entering the platform transact), so there is only the extensive margin, 2) consumers are assumed to know all their preferences before visiting the platform, so there are no hold-up issues, 3) they address different research questions. As a result, this literature cannot address the questions we aim to answer.

Very recently Crawford et al. empirically analyze quality provision by local monopolist cable networks in the US. Using counterfactual analysis they document that cable networks mostly over-provide quality (offer more expensive channels in their bundles) compared to the socially optimal level. The authors argue that this over-provision result is surprising given theoretical predictions of Mussa and Rosen [1978] and Maskin and Riley [1984] that the monopolist provides right amount of quality for high types and under-provides quality for low types. They explain the over-provision result by the fact that exogenous alternative option to cable tv providers, satellite tv provider, offering high quality products. Our paper illustrates that both under-provision or over-provision of variety by the monopolist are possible and conditions of when which type of distortion arises depends on properties of consumers taste distribution for products. We also illustrate the importance of consumers search costs to discover their tastes for these conditions.

2 Benchmark: Multiproduct monopolist

In the benchmark we analyze a multiproduct monopolist selling n symmetrically differentiated products when consumers face no search costs to discover their tastes for the products. This is a useful benchmark since, as we argue below, the multiproduct monopolist's problem of choosing price/quantity for its products and the total number of products to offer (variety provision) is analogous to a monopolist's problem of choosing quantity and quality of one product, like in Spence [1975].

There is mass one of consumers. Each product has the same fixed cost of development, K, and unit cost per quantity, c. Let $D(p, n)$ denote the per-product demand when there are n symmetric variants each priced at p. The multiproduct monopolist's profit is then

$$\Pi(p, n) = (p - c)X(p, n) - nK,$$

where the total demand is $X(p, n) = nD(p, n)$. We can alternatively invert the demand to obtain $P(x, n)$ and express the profit in terms of the total quantity and variety:

$$\Pi(x, n) = [P(x, n) - c]x - nK.$$

The multiproduct monopolist chooses variety, n, and total quantity of sales, x, to maximize its profit. We assume that the second-order conditions of the monopolist's problem hold:

$$\text{A1. } \frac{\partial^2\Pi}{\partial x^2} < 0 \text{ , A2. } \frac{\partial^2\Pi}{\partial n^2} < 0 \text{ , A3. } \frac{\partial^2\Pi}{\partial n^2}\frac{\partial^2\Pi}{\partial x^2} - (\frac{\partial^2\Pi}{\partial x \partial n})^2 > 0$$

Observe that this problem is analogous to a monopolist's quality provision and price choice in Spence [1975]. Consumer surplus is defined as

$$CS(p, n) = \int_p^\infty X(t, n)dt$$

or alternatively

$$CS(x,n) = \int_0^x P(t,n)dt - P(x,n)x$$

We assume that consumer surplus is decreasing in price p (or increasing in total quantity x), increasing and concave in variety n, respectively:

A4. $\frac{\partial CS(p,n)}{\partial p} < 0$ (or $\frac{\partial CS(x,n)}{\partial x} > 0$), A5. (i) $\frac{\partial CS(p,n)}{\partial n} > 0$, and (ii) $\frac{\partial^2 CS(p,n)}{\partial n^2} < 0$.

These assumptions are natural and hold in commonly used utility specifications. Intuitively, when the firm offers more variants, consumers find a better match to their tastes (A5.i) and more variety gives extra benefits at a decreasing rate (A5.ii) as better (average) matches generate decreasing returns.

The total welfare is the sum of consumer surplus and the firm's profit:

$$W(p,n) = CS(p,n) + \Pi(p,n) \text{ or } W(x,n) = CS(x,n) + \Pi(x,n).$$

Spence [1975] compares the monopolist's provision of quality to the social planner's provision of quality at a given total quantity. Following Spence's analysis, we compare the multiproduct monopolist's variety choice with the social planner's at a given quantity. The thought experiment in the context of the multiproduct monopolist is the following. For a given total quantity of sales, what is the optimal combination of different versions of the product for the monopolist. We obtain modified results of Spence's Proposition 1 after replacing quality by variety:

Lemma 1 *At a given total quantity the multiproduct monopolist under-provides variety compared to the socially optimal level when* $\frac{\partial^2 P}{\partial x \partial n} < 0$. *The monopolist over-provides variety when* $\frac{\partial^2 P}{\partial x \partial n} > 0$. *The monopolist's chooses the socially optimal number of variants at a given total quantity when* $\frac{\partial^2 P}{\partial x \partial n} = 0$.

The proof of Lemma 1 is straightforward and follows the same steps as in Spence. If $\frac{\partial P}{\partial x \partial n} < 0$ then consumer surplus increases in variety more than the increase in the firm's profit from more variety: $\int_0^x \frac{\partial P(t,n)}{\partial n} dt > \frac{\partial P(x,n)}{\partial n} x$, and so total welfare increases by increasing the number of variants above the monopolist's choice. If $\frac{\partial P}{\partial x \partial n} > 0$ then consumer surplus increases in variety less than the increase in the firm's profit from higher variety: $\int_0^x \frac{\partial P(t,n)}{\partial n} dt < \frac{\partial P(x,n)}{\partial n} x$, and so total welfare increases by decreasing the number of variants below the monopolist's choice. If $\frac{\partial P}{\partial x \partial n} = 0$, consumer surplus and the firm's profit are maximized at the same level of variety for a given total quantity.

The multiproduct monopolist's optimal quantity and variety is the solution to the following optimality conditions:

$$\begin{aligned} \frac{\partial \Pi}{\partial x} &= P(x,n) - c + \frac{\partial P}{\partial x} x = 0, \\ \frac{\partial \Pi}{\partial n} &= \frac{\partial P}{\partial n} x - K = 0. \end{aligned} \tag{1}$$

By applying the Implicit Function Theorem to the first condition we derive how the equilibrium level of total quantity changes with variety:

$$\frac{dx^*}{dn} = -\frac{\frac{\partial P}{\partial n} + \frac{\partial^2 P}{\partial x \partial n} x}{\frac{\partial^2 \Pi(x,n)}{\partial x^2}}. \tag{2}$$

The denominator of the latter derivative is negative by the second-order condition (A1) and so the sign of the numerator determines whether the total quantity increases in variety. We would expect in general that more variety shifts the total demand upwards, which we assume thereafter:

$$\text{A6. } \frac{\partial P}{\partial n} + \frac{\partial^2 P}{\partial x \partial n} x > 0.$$

Let us define the second-best optimal variety of the planner, that is, the planner's optimal variety constrained by the monopolist's choice of total quantity and is the solution to

$$\frac{dW(x^*,n)}{dn} = \frac{\partial CS(x^*,n)}{\partial n} + \frac{\partial \Pi(x^*,n)}{\partial n} + [\frac{\partial CS(x^*,n)}{\partial x^*} + \frac{\partial \Pi(x^*,n)}{\partial x^*}]\frac{dx^*}{dn} = 0.$$

If we evaluate the latter derivative at the monopolist's optimal variety we obtain:

$$\frac{dW(x^*,n^*)}{dn} = \frac{dCS(x^*,n^*)}{dn}. \tag{3}$$

since at the monopolist's optimal variety and quantity we have $\frac{\partial \Pi(x^*,n^*)}{\partial n} = \frac{\partial \Pi(x^*,n^*)}{\partial x^*} = 0$. We thereby illustrate how the monopolist's choice of variety distorts welfare compared to the second-best, where the planner can control variety and is constrained with the monopolist's quantity:

Lemma 2 *The monopolist under-provides variety compared to the second-best variety if consumer surplus evaluated at the equilibrium quantity and variety increases in variety,* $\frac{dCS(x^*,n^*)}{dn} > 0$. *Conversely, the monopolist over-provides variety compared* *to the second-best level if consumer surplus decreases in variety,* $\frac{dCS(x^*,n^*)}{dn} < 0$*. The monopolist chooses the second-best optimal variety if the consumer surplus is constant in variety,* $\frac{dCS(x^*,n^*)}{dn} = 0$*.*

We are now ready to illustrate how under-/over-provision of variety in Spence terms (i.e., at a given total quantity) compares to under-/over-provision of variety with respect to the second-best:

Lemma 3 *1. When the monopolist provides optimal variety or under-provides variety in Spence terms (i.e., at a given total quantity), this implies under-provision of variety compared to the second-best optimal.*

2. When the monopolist over-provides variety in Spence terms, this implies over-provision of variety compared to the second-best optimal if the consumer surplus reduction due to too much variety is higher than the consumer surplus reduction due to the monopolist's quantity distortion:

$$\frac{\partial P(x^*,n^*)}{\partial n}x^* - \int_0^{x^*} \frac{\partial P(t,n^*)}{\partial n}dt > -x^* \frac{\partial P(x^*,n^*)}{\partial x^*}\frac{dx^*}{dn}.$$

Otherwise, the monopolist under-provides variety compared to the second-best.

Proof. We first derive how the consumer surplus changes in variety at the equilibrium level of quantity chosen by the monopolist:

$$\frac{dCS(x^*,n)}{dn} = \int_0^{x^*} \frac{\partial P(t,n)}{\partial n}dt - \frac{\partial P(x^*,n)}{\partial n}x^* - x^* \frac{\partial P(x^*,n)}{\partial x^*}\frac{dx^*}{dn}. \tag{4}$$

The monopolist provides the optimal variety at a given quantity if the consumer surplus increases in variety as much as the monopolist's revenue change from more variety: $\int_0^x \frac{\partial P(t,n)}{\partial n}dt = \frac{\partial P}{\partial n}x$, that is, when $\frac{\partial^2 P}{\partial n \partial x} = 0$. In that case, we have $\frac{dCS(x^*,n^*)}{dn} > 0$ (see equation (4)) since $\frac{dx^*}{dn} > 0$ (by (A6)) and the demand is downward sloping: $\frac{\partial P}{\partial x} < 0$. The finding that $\frac{dCS(x^*,n^*)}{dn} > 0$ and equation (3) together imply that $\frac{dW(x^*,n^*)}{dn} > 0$, and thereby that there is under-provision of variety compared to the second-best.

The monopolist under-provides variety at a given quantity if consumer surplus increases in variety more than the monopolist's revenue change from variety: $\int_0^x \frac{\partial P(t,n)}{\partial n}dt > \frac{\partial P(x,n)}{\partial n}x$, that is, when $\frac{\partial^2 P}{\partial n \partial x} < 0$. In that case, we have $\frac{dCS(x^*,n^*)}{dn} > 0$ (see equation

(4)) since $\frac{dx^*}{dn} > 0$ (by (A6)) and $\frac{\partial P}{\partial x} < 0$. Thus, there is under-provision of variety compared to the second-best.

The monopolist over-provides variety at a given quantity if consumer surplus increases in variety less than the monopolist's revenue change from variety: $\int_0^x \frac{\partial P(t,n)}{\partial n} dt < \frac{\partial P(x,n)}{\partial n} x$, that is, when $\frac{\partial^2 P}{\partial n \partial x} > 0$. In that case, we have $\frac{dW(x^*,n^*)}{dn} < 0$ if the discrepancy between the direct consumer surplus effect of variety (at a given quantity) dominates the effect of variety on the total quantity: $\frac{\partial P(x^*,n^*)}{\partial n} x^* - \int_0^{x^*} \frac{\partial P(t,n^*)}{\partial n} dt > -x^* \frac{\partial P(x^*,n^*)}{\partial x^*} \frac{dx^*}{dn}$. Otherwise, we have $\frac{dW(x^*,n^*)}{dn} > 0$.

■

Intuitively, when the monopolist provides right amount of variety at a given total quantity, at the second-best optimum the planner wants to offer more variety to compensate for the quantity reduction due to the monopolist's markup. Similarly, the second-best optimum calls for increasing variety when the monopolist under-provides variety at a given total quantity. On the other hand, the second-best optimum trades-off over-provision of variety and under-provision of quantity when the monopolist provides too much variety at a given total quantity. This trade-off might imply that the planner wants to reduce variety (over-provision of variety) at the second best optimum if the consumer surplus reduction due to too much variety is more important than the consumer surplus reduction due to the restricted total quantity.

Subject to the monopolist's choice of quantity (or price), the net effect of more variety on consumer surplus is not straightforward due to two counter-acting effects of variety: When the monopolist provides more variety, the direct effect of this on the consumer surplus is positive, whereas more variety implies that the monopolist charges higher prices. Thus, there is negative indirect effect of variety on the consumer surplus. Below we will illustrate that the net effect of variety on consumer surplus depends on demand specification (preferences).

2.1 Examples

2.1.1 Symmetric Multinominal Logit (MNL)

Suppose that each buyer gets utility u_i from purchasing product i, $i = 1, 2, .., n$:

$$u_i = v - p_i + \mu \epsilon_i, \tag{5}$$

where v denotes the unit consumption value, p_i denotes the price of product i, ϵ_i is the taste parameter which is assumed to be i.i. double exponentially distributed across products, and product differentiation is measured by parameter μ , which is assumed to be positive. We allow for (exogenous) outside option for buyers by assuming that a buyer gets u_0 if she does not buy any of the n products: $u_0 = v_0 + \epsilon_0$, where v_0 denotes the value of the outside good, the taste for the outside good, ϵ_0, is assumed to be i.i. double exponentially distributed along with the ϵ_i. For simplicity, assume that $v_0 = 0$. Under these assumptions the purchase probability for product i is given by [Anderson et al., 1992]:

$$\mathbb{P}_i(p_i, p_{-i}, n) = \frac{\exp((v-p_i)/\mu)}{\sum_{j=1}^{n} \exp((v-p_j)/\mu) + 1}.$$

Product i's demand is equal to $\mathbb{P}_i$. The consumer surplus is equal to the expected consumption utility:[2]

$$CS = \mu ln \left(\sum_{j=1}^{n} \exp(\frac{v-p_j}{\mu}) + 1 \right).$$

When each product is priced at p, the demand per product will be:

$$\mathbb{P}(p, n) = \frac{\exp((v-p)/\mu)}{n \exp((v-p)/\mu) + 1}. \tag{6}$$

and the consumer surplus will be

$$CS(p, n) = \mu ln \left(n \exp(\frac{v-p}{\mu}) + 1 \right). \tag{7}$$

The total quantity is then

$$X(p, n) = n\mathbb{P}(p, n) = \frac{n \exp((v-p)/\mu)}{n \exp((v-p)/\mu) + 1}. \tag{8}$$

We invert the total demand (at symmetric prices) to obtain the inverse demand :

$$P(x, n) = v + \mu[In(n) + In(1-x) - In(x)] \tag{9}$$

[2]See Anderson et al. [1992], p. 231, for the derivation of the expected utility from consumption in the MNL model with outside good.

Since the inverse demand is additively separable in variety and quantity, we have $\frac{\partial^2 P}{\partial x \partial n} = 0$.

Corollary 1 *When each product's demand is given by the symmetric Multinominal Logit (MNL) demand (6), the equilibrium levels of the total quantity, consumer surplus, and price are all increasing in variety.*

Proof. Recall that the monopolist's profit is

$$\Pi(x, n) = (P(x, n) - c)x - nK.$$

where the inverse demand, $P(x, n)$, is given by equation (9). We first prove that the second-order conditions (A1-A3) of the monopolist's problem hold for the case of symmetric Multinominal Logit demand. We derive the second-order derivatives of the monopolist's profit with respect to quantity and variety:

$$\frac{\partial^2 \Pi}{\partial x^2} = 2\frac{\partial P}{\partial x} + \frac{\partial^2 P}{\partial x^2}x = \frac{\partial P}{\partial x}(1 + \frac{x}{1-x}) < 0, \tag{10}$$

$$\frac{\partial^2 \Pi}{\partial n^2} = \frac{\partial^2 P}{\partial n^2}x = -\frac{\mu}{n^2}x < 0, \tag{11}$$

$$\frac{\partial^2 \Pi}{\partial x \partial n} = \frac{\partial P}{\partial n} = \frac{\mu}{n}. \tag{12}$$

The first inequality (A1) holds since $\frac{\partial P}{\partial x} = -\mu(\frac{1}{1-x} + \frac{1}{x}) < 0$ and $1 + \frac{x}{1-x} > 0$, given that $x < 1$. The second inequality implies that (A2) holds. Furthermore, (A3) holds since

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial x^2}\frac{\partial^2 \Pi}{\partial n^2} - (\frac{\partial^2 \Pi}{\partial x \partial n})^2 &= \frac{\mu^2}{n^2}(x(1 + \frac{x}{1-x})(\frac{1}{1-x} + \frac{1}{x}) - 1) \\ &= \frac{\mu^2}{n^2}(\frac{1}{(1-x)^2} - 1) > 0.\end{aligned}$$

Hence, the monopolist's optimal variety and quantity is the solution to the first-order conditions given in (1).

The total equilibrium quantity increases in variety, so (A6) holds:

$$\begin{aligned}\frac{dx^*}{dn} &= -\frac{\frac{\partial P}{\partial n}+\frac{\partial^2 P}{\partial x \partial n}x}{\frac{\partial^2 \Pi}{\partial x^2}}, \\ &= -\frac{\frac{\mu}{n}}{\frac{\partial^2 \Pi(x,n)}{\partial x^2}} > 0, \end{aligned} \tag{13}$$

since $\frac{\partial^2 P}{\partial x \partial n} = 0$ and the denominator is negative as we showed above. Consumer surplus at the equilibrium quantity increases in variety:

$$\begin{aligned}\frac{dCS(x^*,n)}{dn} &= \int_0^{x^*} \frac{\partial P(t,n)}{\partial n}dt - \frac{\partial P(x^*,n)}{\partial n}x^* - x^* \frac{\partial P(x^*,n)}{\partial x^*}\frac{dx^*}{dn} \\ &= -x^* \frac{\partial P(x^*,n)}{\partial x^*}\frac{dx^*}{dn} > 0.\end{aligned}$$

since the demand is downward sloping, $\frac{\partial P(x,n)}{\partial x} < 0$, and the total equilibrium quantity increases in variety, $\frac{dx^*}{dn}$ as shown previously. We finally show that the equilibrium price is also increasing in variety:

$$\begin{aligned}\frac{dP(x^*,n)}{dn} &= \frac{\partial P(x^*,n)}{\partial n} + \frac{\partial P(x^*,n)}{\partial x^*}\frac{dx^*}{dn}, \\ &= \frac{\mu}{n} - \frac{\partial P(x^*,n)}{\partial x^*}\frac{\frac{\mu}{n}}{\frac{\partial^2 \Pi}{\partial x^2}} \\ &= \frac{\mu}{n}\left(1 - \frac{1}{1+\frac{x^*}{1-x^*}}\right) = \frac{\mu}{n}x^* > 0\end{aligned}$$

where for the first equality we used (13) and for the second equality we used (10) at x^*. ■

We thus show that in the symmetric MNL model the monopolist sets a higher price for each product when it offers more differentiated products. At the price chosen by the monopolist the total demand and the consumer surplus increase in variety. These imply that offering one more differentiated good will make consumers better-off. This illustrates under-provision with respect to the second-best optimal variety. Given that $\frac{\partial^2 P}{\partial x \partial n} = 0$, using Lemmas 1 and 3 we prove the main result in the symmetric MNL model:

Proposition 1 *When each product's demand is given by the symmetric Multinominal Logit demand (6), the monopolist chooses the socially optimal variety at a given* *quantity (no Spence distortion), but under-provides variety compared to the second-best optimum.*

It is important to emphasize that with the symmetric MNL demand the multi-product monopolist under-provides variety compared to the socially optimal variety subject to the monopolist's pricing (the second-best optimum). This result is particularly important for empirical work that uses MNL demand to analyze optimal variety provision of a multi-product firm. Due to the structure of MNL demand, the monopolist always under-provides variety. We will see below that this result is valid also when products are asymmetric in utility they generate for consumers as long as the demand structure is MNL. On the other hand, this result might be different for other demand specifications as we will see below.

2.1.2 Asymmetric MNL

Now we consider the case of MNL model where products are asymmetric in their consumption utility, so the net utility of buying product i is

$$u_i = v_i - p_i + \mu\epsilon_i, \tag{14}$$

where v_i denotes consumption utility from product i, p_i denotes product i's price, ϵ_i is the random taste shock, and μ measures differentiation between products, and the utility of not buying any product is $u_0 = \epsilon_0$.

We assume that random taste shocks (ϵ_i s and ϵ_0) are double exponentially distributed. The demand for product i is then given by asymmetric MNL:

$$\mathbb{P}_i = \frac{exp\left(\frac{v_i - p_i}{\mu}\right)}{\sum_{j=1}^{n} exp\left(\frac{v_j - p_j}{\mu}\right) + 1}. \tag{15}$$

We will firstly illustrate that in equilibrium the multiproduct monopolist sets the same markup on each product. Consider the profit of the monopolist:

$$\Pi(p_1, p_2, .., p_n, n) = \sum_{j=1}^{n} (p_j - c)\mathbb{P}_j - nK.$$

The monopolist's first-order condition with respect to p_i is

$$\frac{\partial \Pi}{\partial p_i} = \mathbb{P}_i + (p_i - c)\frac{\partial \mathbb{P}_i}{\partial p_i} + \sum_{k \neq i}(p_k - c)\frac{\partial \mathbb{P}_k}{\partial p_i} = 0. \tag{16}$$

Using the properties of MNL we derive

$$\begin{aligned} \frac{\partial \mathbb{P}_i}{\partial p_i} &= -\frac{\mathbb{P}_i(1-\mathbb{P}_i)}{\mu}, \\ \frac{\partial \mathbb{P}_k}{\partial p_i} &= \frac{\mathbb{P}_k \mathbb{P}_i}{\mu} \text{ for } k \neq i. \end{aligned}$$

Replacing the latter derivatives into the monopolist's first-order condition with respect to p_i proves that

Lemma 4 *When the demand for each product is given by asymmetric MNL demand, the multiproduct monopolist sets the same markup, m, for each product:*

$$m = p_i - c = \mu + \sum_{j=1}^{n}(p_j - c)\mathbb{P}_j \textit{ for all } i.$$

The monopolist's optimal per-product markup is then $m^* = \frac{\mu}{1-x}$, where x denotes the total demand for the monopolist: $x = \sum_{j=1}^{n} \mathbb{P}_j$.

Next we will show that when the markup is the same for all products, the total demand for the monopolist shifts in parallel when the monopolist sells more products, and thus the monopolist provides socially optimal level of variety at a given total quantity level, i.e., there is no Spence distortion, in the asymmetric MNL model.

Let's write the total demand for the monopolist as a function of per-product markup, m:

$$x = \frac{\sum_{j=1}^{n} \exp\left(\frac{v_j - m - c}{\mu}\right)}{\sum_{j=1}^{n} \exp\left(\frac{v_j - m - c}{\mu}\right) + 1},$$

which we can re-write as

$$x = \frac{\sum_{j=1}^{n} \frac{\exp\left(\frac{v_j - c}{\mu}\right)}{\exp\left(\frac{m}{\mu}\right)}}{\sum_{j=1}^{n} \frac{\exp\left(\frac{v_j - c}{\mu}\right)}{\exp\left(\frac{m}{\mu}\right)} + 1},$$

and then by taking $\exp\left(\frac{m}{\mu}\right)$ outside the summation both in the numerator and in the denominator, and by cancelling it, we have

$$x = \frac{\sum_{j=1}^{n} \exp\left(\frac{v_j - c}{\mu}\right)}{\sum_{j=1}^{n} \exp\left(\frac{v_j - c}{\mu}\right) + \exp\left(\frac{m}{\mu}\right)}.$$

Using the latter equality we write the per-product markup as a function of the total quantity:

$$m(x, n) = \mu \ln\left(\frac{1 - x}{x} \sum_{j=1}^{n} \exp\left(\frac{v_j - c}{\mu}\right)\right)$$

Now consider how the markup changes when the total demand changes:

$$\frac{dm}{dx} = -\frac{\mu}{x(1 - x)},$$

which illustrates that the per-product markup is decreasing in the total quantity and also that the inverse demand (captured by the per-product markup plus the marginal cost) shifts in parallel when the total variety, n, increases: $\frac{d^2 m}{dxdn} = \frac{d^2 P}{dxdn} = 0$. Thus, we extend the result of Proposition 1 to the case of asymmetric MNL:

Proposition 2 *When the demand for each product is given by asymmetric Multinominal Logit demand (15), the multiproduct monopolist chooses the socially optimal variety at a given total quantity (no Spence distortion), but under-provides variety compared to the second-best optimum.*

The second part of the proposition is implied by Lemma 3. Observe that Lemma 3 applies in the current case of the asymmetric MNL. To see this we can simply replace the inverse demand by the per-product markup plus the marginal cost: $P(x, n) = m(x, n) + c$.

2.1.3 Discrete choice model with deterministic outside option (DCM)

Similar to the symmetric MNL model, each buyer gets utility u_i from purchasing product i, $i = 1, 2, .., n$:

$$u_i = -p + \mu \epsilon_i, \tag{17}$$

where p is the price of each product (using symmetry), ϵ_i is the taste parameter, and product differentiation is measured by parameter μ, which is assumed to be positive.

Different from the MNL analysis we set the deterministic part of the utility at zero, $v = 0$, (for simplicity), allow for more general distribution of tastes, and assume that the outside option for buyers, that is, the utility of not buying any of the n products is deterministic and set at zero: $u_0 = 0$.[3] Assume that ϵ_i is i.i.d with c.d.f $F(\cdot)$ and p.d.f. $f(\cdot)$. Under these assumptions the probability of purchasing the outside good is

$$Prob(-p + \mu\epsilon_i < 0 \text{ for all } i) = F^n(\frac{p}{\mu}). \tag{18}$$

The total demand for the monopolist's products is the probability of not buying the outside good: $X(p,n) = 1 - F^n(\frac{p}{\mu})$, and the inverse demand is then

$$P(x,n) = \mu F^{-1}((1-x)^{\frac{1}{n}}). \tag{19}$$

We derive the price with respect to the total quantity and variety:

$$\frac{\partial P}{\partial x} = -\frac{\mu(1-x)^{\frac{1-n}{n}}}{nf(\frac{p}{\mu})}, \tag{20}$$

$$\frac{\partial P}{\partial n} = -\frac{\mu(1-x)^{\frac{1}{n}} In(1-x)}{n^2 f(\frac{p}{\mu})}. \tag{21}$$

Observe that the price is increasing in variety: $\frac{\partial P}{\partial n} > 0$, since $In(1-x) < 0$ given that the total demand is positive and less than 1: $x \in (0,1)$. We furthermore derive the cross-derivative of the price with respect to the total quantity and number of products:

$$\frac{\partial^2 P}{\partial x \partial n} = -\frac{\mu\frac{\partial(1-x)^{\frac{1-n}{n}}}{\partial n} nf(\cdot) - \mu(1-x)^{\frac{1-n}{n}}[f(\cdot) + nf'(\cdot)\frac{1}{\mu}\frac{\partial P}{\partial n}]}{n^2 f^2(\cdot)}, \tag{22}$$

$$= \frac{\mu(1-x)^{\frac{1-n}{n}}[\frac{In(1-x)}{f(\cdot)}(f^2(\cdot) - f'(\cdot)F(\cdot)) + f(\cdot)]}{n^2 f^2(\cdot)} \tag{23}$$

where the latter is obtained by replacing the equality of $\frac{\partial P}{\partial n}$ (equation 21) into equation (22). Since $1 - x = F^n(\cdot)$, we have $In(1-x) = nIn(F(\cdot))$. We replace this into the latter equality and define $g(z) \equiv In(F(z))(f^2(z) - f'(z)F(z)) + f^2(z)$. Observe that

[3] Deterministic outside option is to simplify the analysis. If we allowed outside option to have a random value, ϵ_0, which is distributed with other epsilons, the analytical solution of the dcm model would not be feasible.

the cross derivative's sign is then determined by the sign of $g(z)$. Define $y(z) = -In(F(z))$ and we rewrite $F(z) = e^{-y(z)}$, so $f(z) = -e^{-y(z)}y'(z) = -F(z)y'(z)$ and $f'(z) = -f(z)y'(z) - F(z)y''(z) = F(z)(y'(z))^2 - F(z)y''(z)$. We then rewrite the condition as $g(z) > 0$ if and only if $-y(z)y''(z) + y'(z)^2 > 0$, that is, $y(z)$ is strictly log-concave or $In(-In(F(z)))$ is strictly concave. The following proposition summarizes our findings so far:

Lemma 5 *Suppose that each product's demand is given by the general discrete choice model (DCM) where the outside option is deterministic (and normalized to zero) and taste shocks are i.i.d with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$. The inverse demand for the multiproduct monopolist is then $P(x,n) = \mu F^{-1}((1-x)^{\frac{1}{n}})$, where x denotes the total quantity and n denotes the number of products (variety). We have (i) $\frac{\partial^2 P}{\partial x \partial n} > 0$ if $-In(F(\cdot))$ is strictly log-concave, (ii) $\frac{\partial^2 P}{\partial x \partial n} = 0$ if $-In(F(\cdot))$ is log-linear, (iii) $\frac{\partial^2 P}{\partial x \partial n} < 0$ if $-In(F(\cdot))$ is strictly log-convex.*

Using Lemmas 1, 3, and 5 we prove the main result in the DCM model:

Proposition 3 *Suppose that each product's demand is given by the general DCM where the outside option is deterministic (and normalized to zero) and taste shocks are i.i.d with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$. If $-In(F(\cdot))$ is strictly log-concave, the multiproduct monopolist over-provides variety at a given quantity. If $-In(F(\cdot))$ is log-linear, the multiproduct monopolist provides right amount of variety at a given quantity and under-provides variety compared to the second-best optimum. If $-In(F(\cdot))$ is strictly log-convex, the multiproduct monopolist under-provides variety at a given quantity and also with respect to the second-best optimum.*

For instance, when tastes are distributed with Extreme Value Type I, $In(F(\cdot))$ is log-linear and so there is no Spence distortion, whereas by Lemma 3, the monopolist under-provides variety compared to the second best optimum. When taste distribution is exponential, $-In(F(\cdot))$ is strictly log-convex, so there is under-provision of variety in Spence terms and under-provision with respect to the second best with exponentially distributed tastes. There might be over-provision both in Spence terms and with respect to the second-best optimum with uniformly distributed tastes. This is the case when the products are horizontally differentiated on the Salop circle. See Appendix E for the illustration of this case.

The monopolist's profit is

$$\Pi(x,n) = (\mu F^{-1}((1-x)^{\frac{1}{n}}) - c)x - nK. \tag{24}$$

The privately optimal quantity x^* and variety n^* are the solution to the following first-order conditions:

$$\frac{\partial \Pi}{\partial x} = \mu F^{-1}((1-x)^{\frac{1}{n}}) - c + x(-\frac{\mu(1-x)^{\frac{1-n}{n}}}{nf(\frac{P(x,n)}{\mu})}) = 0 \tag{25}$$

$$\frac{\partial \Pi}{\partial n} = x(-\frac{\mu In(1-x)(1-x)^{\frac{1}{n}}}{n^2 f(\frac{P(x,n)}{\mu})}) - K = 0 \tag{26}$$

Consumer surplus is

$$CS = \int_0^x \mu F^{-1}(1-u)^{\frac{1}{n}} du - \mu F^{-1}(1-x)^{\frac{1}{n}} x \tag{27}$$

In Lemma 2 we show that consumer surplus increases in variety at the monopolist's choice of quantity when there is under-provision of variety compared to the second best. Moreover, Proposition 3 illustrates conditions on taste distribution under which this will be the case. Combination of these results give us the following corollary:

Corollary 2 *Suppose that each product's demand is given by the general DCM where the outside option is deterministic (and normalized to zero) and taste shocks are i.i.d with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$. If $-In(F(\cdot))$ is log-linear (e.g., Extreme Value Type I) or strictly log-convex (e.g., Exponential distribution), the consumer surplus increases in variety at the quantity (or price) chosen by the monopolist and the monopolist under-provides variety compared to the second-best optimum.*

We have three key takeaways from these results so far: 1) Consumers preferences for variety (how tastes for different products are distributed) are crucial to determine whether the multiproduct monopolist distorts variety and which way this distortion goes, 2) When demand for products is given by commonly used Multinomial Logit, the monopolist provides right variety at a given quantity (no Spence distortion), but under-provides variety compared to the second-best (optimal variety constrained by the monopolist's pricing). This result is true both when products are symmetrically differentiated and also when they are asymmetrically differentiated, 3) For more general distribution of tastes in discrete choice demand models, whether and which way the monopolist distorts variety depends on log-log concavity properties of the distribution function, that is, whether $-In(F(\cdot))$ is strictly log-concave/log-linear/strictly log-convex. When tastes are exponentially distributed ($-In(F(\cdot))$ is strictly log-convex), the monopolist under-provides variety compared to Spence benchmark and also compared to the second-best optimum. When tastes are Extreme Value Type I distributed ($-In(F(\cdot))$ is log-linear), the monopolist provides right amount of variety compared to Spence benchmark and under-provides variety compared to the second-best optimum. When products are differentiated on the Salop circle, the monopolist over-provides variety compared to the second-best optimum and also compared to the first-best optimum.

We furthermore illustrate that for different specifications of linear demand under-/over-provision of variety by the monopolist is possible. We also show that for the CES demand the monopolist provides too little variety compared to the second-best as well as in Spence terms. The results of different demand specifications and which type of variety distortion the monopolist's variety choice will imply under these specifications are summarized in Table 1 below.

Until so far we assume that consumers know their tastes for products. In many settings it might be the case that consumers can learn about their tastes only after incurring some costs. For instance, consumers need to visit stores and inspect products to discover what they like and which product they prefer. Alternatively, consumers need to spend time online by comparing offerings of different sellers for the product they are interested in. To capture such widely observed situations we next extend the benchmark model by introducing search costs for consumers to discover their tastes.

3 Multiproduct monopolist with costs to discover tastes

Suppose that consumers have to incur an intrinsic search/travel cost, τ, to visit the monopolist's shop and consumers can discover their tastes for products fully once they visit the shop. Such a change in the information structure will lead to two distinct demand margins: 1) Extensive margin where consumers decide whether

Table 1: Demand Specifications* and Implied Variety Distortions by the Monopolist

Spence Optimal Under-provision wrt SB	**Under-provision in Spence Under-provision wrt SB**	**Over-provision in Spence**
Multinominal Logit $\mathbb{P}_i = \frac{exp\left(\frac{v_i-p_i}{\mu}\right)}{\sum_{j=1}^{n} exp\left(\frac{v_j-p_j}{\mu}\right)+1}$	CES $q = \left(\frac{\beta\theta n^{\theta(1-\beta)}}{p}\right)^{\frac{1}{1-\beta\theta}}$ where $0 < \beta\theta, \beta < 1$	Spence-Dixit-Vives $q = \frac{(\delta-p)\mu N}{(1+(n-1)\sigma)\delta}$ where $0 < \sigma < 1$
Shubik-Levitan $q = \frac{\alpha-\beta p}{n}$	Singh-Vives Häcker (2000) $q = \frac{\alpha-p}{\gamma(n-1)+1}$ if $\gamma > 1$ close substitutes	Singh-Vives Häcker (2000) $q = \frac{\alpha-p}{\gamma(n-1)+1}$ if $0 < \gamma < 1$ very differentiated
Discrete Choice if $-In(F(\cdot))$ is log-linear e.g., EV Type I	Discrete Choice if $-In(F(\cdot))$ is str. log-convex, e.g., Exponential	Discrete Choice if $-In(F(\cdot))$ is str. log-concave, e.g., Uniform for high x
		Vicrey-Salop circle also wrt SB/FB.

$*$: Per-product demand is denoted by $\mathbb{P}_i$ in the Multinominal Logit model and denoted by q in other demand specifications.

to visit the shop and incur costs of visiting to discover their tastes for products, 2) Intensive margin where those consumers who visit the shop decide whether to purchase a product and which one to purchase. Consumers differ in their participation costs such that τ is distributed over $[0, \overline{\tau}]$ with the probability density function $h(\tau)$ and cumulative distribution function $H(\tau)$. We assume that $h(\tau)$ is a continuous and log-concave function. Log-concavity of $h(\tau)$ implies the log-concavity of $H(\tau)$ which in turn implies that $h(\tau)/H(\tau)$ is decreasing [Bagnoli and Bergstrom, 1989]. We also assume that the distribution of search costs, τ, is independent of distribution of tastes for products, ϵ_is. Intuitively, this assumption states that consumers with high search costs (for instance, due to high opportunity cost of time) value variety in a way that is not systematically different from consumers with low search costs (random differences are allowed). Let $\tilde{\tau}$ denote the marginal consumer who is indifferent between visiting the shop or not. All consumers with types $\tau \leq \tilde{\tau}$ will then visit the shop and so consumer demand for participation is given by $H(\tilde{\tau})$. We will define $\tilde{\tau}$ below.

Let $V(p, n)$ denote a consumer's indirect utility from visiting the shop, that is, choosing her favourite product among n variants when each variant is priced at p. We have $V(p,n) = \int_p^\infty X(t,n)dt$, where $X(p,n)$ is the total demand per consumer visiting the shop (intensive margin).

Observe that in the benchmark analysis there was only intensive margin since all consumers were visiting the shop by construction. This case corresponds to situations where there are no costs of discovering tastes or very low costs so that all consumers find it optimal to visit the shop in equilibrium. Thus, the indirect utility from visiting the shop in the current setup, $V(p,n)$, is equal to consumer surplus of the benchmark model (without extensive margin). We therefore keep assumptions A4 and A5 for $V(p,n)$:

A4'. $\frac{\partial V(p,n)}{\partial p} < 0$, A5'. (i) $\frac{\partial V(p,n)}{\partial n} > 0$, and (ii) $\frac{\partial^2 V(p,n)}{\partial n^2} < 0$.

The marginal consumer at the participation margin is the one with cost equal to the indirect utility: $\tilde{\tau} = V(p,n)$. Those consumers with costs less than $\tilde{\tau}$ will visit the shop. Thus, consumer participation demand (extensive margin) is equal to $H(\tilde{\tau}) = H(V(p,n))$. The total demand is then the product of the intensive margin and the extensive margin: $\tilde{X}(p,n) = X(p,n)H(V(p,n))$.

Total consumer surplus is the sum of the indirect utility of those consumers who visit the shop minus their participation costs:

$$CS(p,n) = \int_0^{\tilde{\tau}} (\tilde{\tau} - \tau)h(\tau)d\tau = \int_0^{V(p,n)} (V(p,n) - \tau)h(\tau)d\tau. \tag{28}$$

The monopolist's profit is the per-product margin times the total demand minus costs of variety:

$$\Pi(p,n) = (p-c)X(p,n)H(V(p,n)) - nK \tag{29}$$

The firm's optimal variety and price is the solution to the following optimality conditions (where we dropped the arguments of the functions for simplicity):

$$\begin{aligned} \frac{\partial \Pi}{\partial p} &= XH + (p-c)[\frac{\partial X}{\partial p}H + Xh\frac{\partial V}{\partial p}] = 0, \\ \frac{\partial \Pi}{\partial n} &= (p-c)[\frac{\partial X}{\partial n}H + Xh\frac{\partial V}{\partial n}] - K = 0. \end{aligned} \tag{30}$$

Observe that by rearranging the terms we can re-write the latter optimality conditions in a more intuitive way, respectively,

$$\begin{aligned}\frac{p^*-c}{p^*} &= \frac{1}{\epsilon_{X,p}+\epsilon_{H,p}},\\ \frac{nK}{HX} &= \epsilon_{X,n}+\epsilon_{H,n}.\end{aligned} \tag{31}$$

where $\epsilon_{X,p} = -\frac{p}{X}\frac{\partial X}{\partial p}$ is the elasticity of the intensive margin with respect to price, $\epsilon_{H,p} = -\frac{p}{H}h\frac{\partial V}{\partial p}$ is the elasticity of the extensive margin with respect to price, $\epsilon_{X,n} = \frac{n}{X}\frac{\partial X}{\partial n}$ is the elasticity of intensive margin with respect to variety and $\epsilon_{H,n} = \frac{n}{H}h\frac{\partial V}{\partial n}$ is the elasticity of the extensive margin with respect to variety. Intuitively, the monopolist's optimal markup ratio is equal to the inverse of the sum of the extensive and intensive margin price elasticities. The monopolist's optimal variety provision trades off the gains at the extensive margin and at the intensive margin against the costs of variety. At the optimal variety the monopolist equates the average cost of variety, $\frac{nK}{HX}$, to the sum of the variety elasticities of intensive and extensive margins.

Total welfare is the sum of the firm's profit and consumer surplus: $W(p,n) = \Pi(p,n) + CS(p,n)$. To see potential deviations from the second-best variety, we calculate the derivative of welfare with respect to variety at the price and variety chosen by the monopolist:

$$\frac{dW(p^*,n^*)}{dn} = \frac{dCS(p^*,n^*)}{dn} = H\frac{dV(p^*,n^*)}{dn}. \tag{32}$$

We thereby prove that Lemma 2 is valid with the extensive margin after we replace consumer surplus by its new expression (equation (28)); new consumer surplus is increasing in the indirect utility from participation, $V(p,n)$, and the indirect utility is equal to the consumer surplus expression that we derived in the benchmark model (with only intensive margin).

3.1 Examples

3.1.1 Symmetric Multinominal Logit (MNL)

Recall from Section 2.1.1 that in the symmetric MNL model per-consumer demand is

$$X(p,n) = n\mathbb{P}(p,n) = \frac{n\exp((v-p)/\mu)}{n\exp((v-p)/\mu)+1}.$$

and the indirect utility from participation is

$$V(p,n) = \mu \ln \left(n \exp(\frac{v-p}{\mu}) + 1 \right).$$

We define $\Omega(p,n) \equiv n \exp((v-p)/\mu)$ and rewrite per-consumer demand and the indirect utility as a function of Ω: $X(\Omega) = \frac{\Omega}{\Omega+1}$ and $V(\Omega) = \mu \ln(\Omega + 1)$. This illustrates that once we fix Ω we determine both intensive and extensive margins. We can therefore express the total demand as a function of Ω:

$$\tilde{X}(\Omega) = \frac{\Omega}{\Omega+1} H(\mu \ln(\Omega+1)). \tag{33}$$

We prove then our main result for the symmetric MNL model with extensive margin:

Proposition 4 *In the model with costs to discover tastes if per-consumer demand for a product is given by symmetric MNL model, the multiproduct monopolist provides optimal variety at a given total quantity, i.e., there is no Spence distortion.*

Proof. We totally differentiate the equality for $\tilde{X}$, equation (33), and obtain

$$d\tilde{X} = \frac{[H(\cdot) + \Omega\frac{\mu}{\Omega+1}h(\cdot)](\Omega+1) - \Omega H(\cdot)}{(\Omega+1)^2}(\frac{\partial\Omega}{\partial n}dn + \frac{\partial\Omega}{\partial p}dp).$$

In the latter equation by setting $d\tilde{X} = 0$, we derive the partial derivative of price with respect to variety:

$$\frac{\partial p(\tilde{x}, n)}{\partial n} = -\frac{\frac{\partial\Omega}{\partial n}}{\frac{\partial\Omega}{\partial p}} = \frac{\mu}{n},$$

where $\Omega = n \exp((v-p)/\mu)$, so $\frac{\partial\Omega}{\partial n} = exp((v-p)/\mu)$ and $\frac{\partial\Omega}{\partial p} = -\frac{n}{\mu}exp((v-p)/\mu)$. Finally, taking the second-order derivative of price with respect to the total demand and variety proves the result: $\frac{\partial^2 p(\tilde{x},n)}{\partial\tilde{x}\partial n} = 0$. Lemma 1 then implies that there is no Spence distortion. ■

To illustrate the idea behind the proof of Proposition 4 let us consider an increase in n by Δn and an increase in p by Δp such that we keep the total demand, $\tilde{X}$, constant. Given that the total demand is the function of Ω only (see equation (33)) keeping $\tilde{X}$ constant is equivalent to keeping Ω constant. But then observe that we keep both extensive margin, $H(V(\Omega))$, and intensive margin, $X(\Omega)$, constant at the same time given that both margins depend only on Ω. This argument proves that an increase in n by Δn at a given total quantity $\tilde{X}$ keeps the total consumer surplus constant (equation (28)), i.e., results in a parallel upward shift of the demand curve, $\frac{\partial^2 P(\tilde{x},n)}{\partial \tilde{x} \partial n} = 0$. Recall that in this case there is no Spence distortion since the marginal consumer and the average consumer value additional variety by the same amount (Lemma 1).

On the other hand, we show that the monopolist under-provides variety compared to the second best in the limit when search costs are not binding, i.e., when nearly all consumers visit the store:

Proposition 5 *In the model with extensive and intensive margins if per-consumer demand for a product is given by MNL model, the multiproduct monopolist under-provides variety compared to the second-best optimum when search costs are not binding so that nearly all consumers visit the store.*

Proof. Recall that in the MNL model with symmetric products the per-product per-consumer demand is

$$\mathbb{P}(p, n) = \frac{\exp((v - p)/\mu)}{n \exp((v - p)/\mu) + 1}.$$

and the indirect utility from visiting the store is

$$V(p, n) = \mu In \left(n \exp((v - p)/\mu) + 1\right).$$

Recall also that the total consumer surplus is

$$CS(p, n) = \int_0^{V(p,n)} (V(p, n) - \tau) h(\tau) d\tau.$$

The profit of the monopolist is then

$$\Pi(p, n) = ((p - c)n\mathbb{P} - nK)H(V(p, n)).$$

The total welfare is then

$$W(p, n) = \Pi(p, n) + CS(p, n).$$

To determine the distortion with respect to the second-best variety, consider the derivative of the welfare with respect to variety at price and variety chosen by the monopolist:

$$\frac{dW(p^*, n^*)}{dn} = \frac{d\Pi}{dp}\frac{dp^*}{dn} + \frac{d\Pi}{dn} + \frac{dCS}{dp}\frac{dp^*}{dn} + \frac{dCS}{dn}.$$

By definition of optimal variety and price chosen by the monopolist we have $\frac{d\Pi}{dp} = \frac{d\Pi}{dn} = 0$ at p^* and n^*. Using the expression of consumer surplus we derive

$$\frac{dW(p^*, n^*)}{dn} = \frac{dCS}{dp}\frac{dp^*}{dn} + \frac{dCS}{dn} = \left[\frac{dV}{dp}\frac{dp^*}{dn} + \frac{dV}{dn}\right] H(V(p^*, n^*))$$

For the MNL we then drive

$$\begin{aligned} \frac{\partial V}{\partial p} &= -n\mathbb{P} \\ \frac{\partial V}{\partial n} &= \mu\mathbb{P} \end{aligned}$$

The monopolist under-provides variety compared to the second best if and only if

$$\frac{dW(p^*, n^*)}{dn} = \left[-n\mathbb{P}\frac{dp^*}{dn} + \mu\mathbb{P}\right] H(V(p^*, n^*)) > 0,$$

that is, if and only if

$$\frac{dp^*}{dn} < \frac{\mu}{n}$$

When search costs were not binding, i.e., when all consumers visited the store, the monopolist would set its price by maximising its profit $\Pi = (p - c)n\mathbb{P} - nK$, which would give

$$p^* - c = \frac{\mu}{1 - n\mathbb{P}}$$

and in that case $\frac{dp^*}{dn} = \mu\mathbb{P}$, which is smaller than $\frac{\mu}{n}$, so satisfying under-provision condition, given that $n\mathbb{P} < 1$. Thus, we would expect this to be the case in the limit when search costs are not binding, i.e, nearly all consumers visiting the store. ■

On the other hand, when search costs are binding, offering more variety has double-dividend: it increases revenue from consumers visiting the store (intensive margin effect) and it also attracts more consumers to the store (extensive margin effect). Given that the latter effect would not exist when search costs were not binding, we expect the monopolist to set a higher price per product when it offers more variety and faces consumers with search costs than the case without search costs. In other words, our claim is that in the MNL demand with search costs we should have $\frac{dp^*}{dn} > \mu\mathbb{P}$. We derive above that with search costs the monopolist's optimal markup is the inverse of the sum of the intensive and extensive margin elasticities:

$$\frac{p^* - c}{p^*} = \frac{1}{\epsilon_X^p + \epsilon_H^p}.$$

This should then suggest that if the extensive margin is very elastic to variety, that is, offering more variety drives a lot of traffic to the store, the monopolist can increase its price significantly by offering more variety. However, how much the monopolist can increase its price should also depend on the elasticity of intensive and extensive margins to the price. For instance, if both margins are very inelastic to price, but very elastic to variety, the monopolist will charge a very high price when it offers more variety. In that case, we might have over-provision of variety with respect to the second-best if $\frac{dp^*}{dn}$ goes above $\frac{\mu}{n}$ in the MNL model.

3.1.2 Discrete Choice Model with deterministic outside option (DCM)

Now we consider the discrete choice model with deterministic outside option, which is studied in Section 2.1.3. Recall that in that model the total demand for the products is the probability of not buying the outside good: $X(p, n) = 1 - F^n(\frac{p}{\mu})$. Now this corresponds to the intensive margin, that is, the total demand per consumer visiting the shop. The expected utility from visiting the shop corresponds to $V(p, n) = \int_p^\infty (1 - F^n(\frac{t}{\mu}))dt$. The demand for participation (extensive margin) is then $H(V(p.n))$. Thus, the total demand is

$$\tilde{X}(p, n) = X(p, n)H(V(p, n)) = [1 - F^n(\frac{p}{\mu})]H\left(\int_p^\infty (1 - F^n(\frac{t}{\mu})\right) dt. \tag{34}$$

Consider the change in n by Δn and the change in p by Δp such that the extensive margin remains constant, that is, $V(p, n)$ is kept constant:

$$\frac{\Delta p}{\Delta n} = -\frac{\partial V/\partial n}{\partial V/\partial p} = -\frac{\int_p^\infty F^n(\frac{t}{\mu}) InF(\frac{t}{\mu})dt}{1 - F^n(\frac{p}{\mu})}. \tag{35}$$

Now we calculate the change in the total margin, $\Delta\tilde{X}$, at new n and p levels:

$$\Delta\tilde{X} = \frac{F^n H\left(\int_p^\infty (1 - F^n(\frac{t}{\mu})dt\right)}{1 - F^n}\Delta n\left(-(1 - F^n)InF + \frac{fn}{F\mu}\int_p^\infty F^n(\frac{t}{\mu})InF(\frac{t}{\mu})dt\right). \tag{36}$$

The sign of $\Delta\tilde{X}$ is given by the sign of the term inside the parentheses. To investigate this sign we change variable by defining $y(z) = -In(F(\frac{z}{\mu}))$. We then have $y' = -\frac{f}{F\mu}$, $F = e^{-y}$, $F^n = e^{-ny}$, $f = -y'e^{-y}$. Using these definitions we rewrite the term inside the parentheses:

$$sign(\Delta\tilde{X}) = sign\left(y(p)(1 - e^{-ny(p)}) - y'(p)\int_p^\infty -ne^{-ny(t)}y(t)dt\right). \tag{37}$$

We rewrite the term inside the integral by multiplying and dividing it by $y'(t)$. We then apply integration by parts and rewrite the integral term as

$$\int_p^\infty -ne^{-ny(t)}y(t)dt = \left[\frac{e^{-ny(t)}y(t)}{y'(t)}\right]_p^\infty - \int_p^\infty e^{-ny(t)}\left(\frac{y(t)}{y'(t)}\right)' dt.$$

Substituting the latter equality into (37) and rearranging terms we obtain

$$sign(\Delta\tilde{X}) \;=\; sign\left(y(p) - y'(p)lim_{t\to\infty}\frac{e^{-ny(t)}y(t)}{y'(t)} + y'(p)\int_p^\infty e^{-ny(t)}\left(\frac{y(t)}{y'(t)}\right)' dt\right).$$

Observe that $y(p) > 0$ and the second term inside the parantheses is zero:

$$-y'(p)lim_{t\to\infty}\frac{e^{-ny(t)}y(t)}{y'(t)} = 0$$

since $y'(p) = -\frac{f(p/\mu)}{F(p/\mu)\mu} < 0$, $lim_{t\to\infty}y(t) = 0$, so $lim_{t\to\infty}e^{-ny(t)} = 1$, and $lim_{t\to\infty}\frac{e^{-ny(t)}y(t)}{y'(t)} = 0$. Note also that if $-InF(z)$ is log-linear, the third term inside the parentheses is zero since then $\left(\frac{y(t)}{y'(t)}\right)' = 0$. In that case, the total quantity of the monopolist increases if it increases its variety and prices while keeping the extensive margin constant. The same is true when $y(z)$ strictly log-convex since then the third term inside the brackets is positive given $\frac{y(z)}{y'(z)}$ is strictly decreasing. Thus, when $-InF(z)$ is log-convex, we show that the total quantity of the monopolist increases if it increases its variety while increasing prices to keep the extensive margin constant. In other words, when

$-InF(z)$ is log-convex, the intensive margin increases if the monopolist increases its variety while increasing prices to keep the extensive margin constant. It must then be the case that the total demand is relatively more elastic to variety changes than price changes compared to the extensive margin. Mathematically,

$$\frac{\epsilon_{\tilde{X},n}}{\epsilon_{\tilde{X},p}} = \frac{\epsilon_{X,n} + \epsilon_{H,n}}{\epsilon_{X,p} + \epsilon_{H,p}} > \frac{\epsilon_{H,n}}{\epsilon_{H,p}}.$$

This in turn implies that when $-InF(z)$ is log-convex, the intensive margin is relatively more elastic to variety changes than price changes compared to the extensive margin:

$$\frac{\epsilon_{X,n}}{\epsilon_{X,p}} > \frac{\epsilon_{H,n}}{\epsilon_{H,p}}.$$

On the other hand, when $-InF(z)$ is sufficiently log-concave, that is, when

$$\frac{y(p)}{y'(p)} + \int_{p}^{\infty} e^{-ny(t)} \left(\frac{y(t)}{y'(t)}\right)' dt > 0 \tag{38}$$

the total quantity of the monopolist decreases, $sign(\Delta\tilde{X}) < 0$. In that case, the intensive margin decreases if the monopolist increases its variety while increasing prices to keep the extensive margin constant. It must then be the case that the total demand and so the intensive margin is relatively less elastic to variety changes than price changes compared to the extensive margin:

$$\frac{\epsilon_{X,n}}{\epsilon_{X,p}} < \frac{\epsilon_{H,n}}{\epsilon_{H,p}}.$$

Proposition 6 *In the general DCM model with deterministic outside option suppose we allow both elastic participation (extensive) margin and transaction (intensive) margin. Let $F(z)$ denote the cdf of consumers' tastes for products. When $-InF(z)$ is log-convex, the intensive margin is relatively more elastic to variety changes than price changes compared to the extensive margin. When $-InF(z)$ is sufficiently log-concave, the opposite is true.*

Now consider the exercise to determine whether and which type of Spence distortion we might have. Suppose that the monopolist increases its variety while increasing prices to keep its total demand constant. In the case where the intensive margin is relatively more elastic to variety than prices compared to the extensive margin, when $-InF(z)$ is log-convex, this change (keeping the total demand constant) must reduce the extensive margin,[4] so reduce per-consumer expected surplus from transactions, $V(p,n)$, which implies that this change reduces the total consumer welfare. This in turn implies that the monopolist over-provides variety at a given quantity (over-provision in Spence terms). In the case where the intensive margin is relatively less elastic to variety than prices compared to the extensive margin, when $-InF(z)$ is sufficiently log-concave, this change (increasing variety and prices while keeping the total demand constant) increases the extensive margin, so increases per-consumer expected surplus from transactions, $V(p,n)$, which implies that this change increases the total consumer welfare. This in turn implies that the monopolist under-provides variety at a given quantity (under-provision in Spence terms).

Proposition 7 *In the general DCM model with deterministic outside option suppose we allow both elastic participation (extensive) margin and transaction (intensive) margin. Let $F(z)$ denote the cdf of consumers' tastes for products. When $-InF(z)$ is log-convex, the monopolist over-provides variety at a given total quantity. When $-InF(z)$ is sufficiently log-concave, the monopolist under-provides variety at a given total quantity.*

This proposition illustrates that when consumers incur search costs to discover their tastes for different products offered by the firm, the direction of distortion introduced

[4]To see why the extensive margin decreases consider Δp and Δn such that $\Delta\tilde{X} = 0$. It must then be the case that Δp and Δn have the same sign since a higher price decreases total demand and more variety increases total demand. Given that total demand is the multiplication of intensive and extensive margins: $\tilde{X} = X.H$, we can write the previous equality in terms of elasticities:

$$\begin{aligned}
\Delta\tilde{X} &= \Delta X.H + X.\Delta H = 0 \\
&= \epsilon_{X,n}.\Delta n.\frac{X}{n}.H - \epsilon_{X,p}.\Delta p.\frac{X}{p}.H + \epsilon_{H,n}.\Delta n.\frac{H}{n}.X - \epsilon_{H,p}.\Delta p.\frac{H}{p}.X = 0 \\
&= HX\epsilon_{X,p}\left(\frac{\epsilon_{X,n}}{\epsilon_{X,p}}\frac{\Delta n}{n} - \frac{\Delta p}{p}\right) + HX\epsilon_{H,p}\left(\frac{\epsilon_{H,n}}{\epsilon_{H,p}}\frac{\Delta n}{n} - \frac{\Delta p}{p}\right) = 0 \qquad (39)
\end{aligned}$$

We show in Proposition 6 that when $-InF(z)$ is log-convex, the intensive margin is relatively more elastic to variety changes than price changes compared to the extensive margin: $\frac{\epsilon_{X,n}}{\epsilon_{X,p}} > \frac{\epsilon_{H,n}}{\epsilon_{H,p}}$. But then equality 39 implies that the first term inside the parentheses must be positive and the second term inside the parentheses must be negative given that the sum of these terms is zero, $H, X, \epsilon_{X,p}, \epsilon_{H,p}, \epsilon_{X,n}, \epsilon_{H,n}, p, n > 0$ and Δn and Δp have the same sign. Thus, we prove that $\Delta X > 0$ and $\Delta H < 0$. In other words, the extensive margin decreases and the intensive margin increases when the firm changes p and n to keep the total demand unchanged (when $-InF(z)$ is log-convex).

by the monopolist variety provision is different compared to the benchmark without search costs. When $-InF(z)$ is strictly log-convex (e.g., exponential distribution), the monopolist over-provides variety at a given total quantity with search costs, whereas the monopolist under-provides variety at a given quantity without search costs. Interestingly, when $-InF(z)$ is log-linear (e.g., Extreme Value Type I), without search costs the monopolist provides right amount of variety at a given quantity, however the monopolist over-provides variety (at a given quantity) with search costs. On the other hand, under-provision of variety by the monopolist (at a given quantity) requires $-InF(z)$ to be sufficiently log-concave with search costs, whereas under-provision of variety happens when $-InF(z)$ is strictly log-convex with search costs.

These results suggest that over-provision of variety by the monopolist becomes more plausible (or under-provision becomes less likely) when consumers face search costs to learn how much they like different products offered by the monopolist. Intuitively, consumers have to incur search costs (e.g., visit the store) to learn how much they actually value each product. Once consumers decide to visit the store and incur these costs, they might end up not purchasing any product of the monopolist if it happens that their highest surplus from consuming a product of the monopolist is less than the value of the outside good. This ex-ante uncertainty about how much surplus consumers would generate from visiting the store, shifts the total demand downward. In order to convince consumers to visit the store, the monopolist might want to offer a larger portfolio of products which increases the consumer expected surplus from being matched to their best product and so increases consumer participation. How more variety affects consumer participation demand (extensive margin) depends on how much the extensive margin changes with variety relative to how much it changes with price vs how much intensive margin changes with variety relative to how much it changes with price. This is because when the firm offers more variety, the direct effect of variety increases consumers' expected surplus from visiting the store, whereas the indirect effect of variety (via higher prices) lowers their expected surplus. Which effect dominates depends on the relative elasticities of extensive and intensive margins. Our previous results illustrate that the relative elasticities of extensive and intensive margins depend on the log-log concavity of the distribution of tastes for variety.

4 Application: E-commerce Platform

In this section we will argue how previous analysis of multiproduct monopolist variety provision with search costs will apply in the context of an online trade platform, like eBay, and help us analyse the optimal variety provision in e-commerce. Now consider the problem of a trade platform which facilitates transactions between buyers and sellers. We will firstly illustrate under which conditions the monopoly trade platform's problem of choosing its fees to sellers will be equivalent to the multiproduct monopolist's problem of choosing its prices and variety (number of products). We will then illustrate how using this equivalence result will provide insights on the variety provision by the trade platform and its comparison to the optimality benchmarks. We furthermore illustrate how the platform's choice of seller contract type affect variety provision to consumers.

E-commerce platforms, like eBay, charge fees (commissions) to sellers and zero fees to buyers. Sellers' incentives to list their products on a platform depend on the platform's seller fees. Seller commissions paid to the platform are variable costs, which they then pass on partially (or fully) to buyers. Sellers' participation increases with the number of buyers visiting the platform because this increases their potential demand. On the other side, buyers mostly find it costly to visit the platform due to search/time costs. They can evaluate how much they like each product once they are on the platform. Such search frictions imply that the number of buyers visiting the platform depends on prices and variety of products buyers expect to find on the platform. Platforms use their contract conditions with sellers to balance demand on both sides, which then determines the level of prices and variety of products provided on platforms, which in turn dictates buyer and seller surpluses.

To capture these important facts of online trade platforms we consider the following framework. There is mass 1 of buyers who is willing to buy one unit of a product on the platform. Buyers have to pay an intrinsic search cost, τ, to enter the platform, but the platform does not charge any fee to buyers.There is buyer heterogeneity in search cost τ such that τ is distributed with the probability distribution function $f(\tau)$ and cumulative density function $F(\tau)$ over a compact interval $[0, \bar{\tau}]$. We assume that $f(\tau)$ is a continuous and log-concave function. Log-concavity of $f(\tau)$ implies the log-concavity of $F(\tau)$ which in turn implies that $f(\tau)/F(\tau)$ is decreasing [Bagnoli and Bergstrom, 1989]. Let $\tilde{\tau}$ denote the marginal buyer who is indifferent between entering the platform or not. All buyers with types $\tau \leq \tilde{\tau}$ will then enter the platform and so buyer demand for participation is given by $F(\tilde{\tau})$. We will define $\tilde{\tau}$ below.

The number of sellers on the platform, n, is endogenously determined by free-entry condition of sellers that we explain below. The platform charges a fee per transaction, w_i, and a listing fee (fixed over transactions), ϕ_i, to seller i. In addition to the platform's fees each seller incurs the marginal cost of c and fixed cost of K.

The timing of the events is the following.

1. The platform sets a unit fee, w_i, and a fixed fee, ϕ_i, to seller i.
2. Sellers decide whether to accept the platform's contract. If so, they list their product on the platform and set its price. Buyers observe the platform's fees and decide whether to enter the platform.
3. Buyers observe products' prices and their valuations of products, and decide which product to purchase (if any).

Let $D_i(p_i, \mathbf{p_{-i}})$ denote the demand for seller i's product on the platform when its price is p_i and the vector of its rivals' prices is $\mathbf{p_{-i}}$. We assume that $D_i(\cdot)$ is symmetric for all sellers, decreasing in its own price and products are imperfect substitutes: $-\partial_{p_i} D_i(\cdot) > \partial_{p_j} D_i(\cdot) > 0$ for any rival j of seller i. We also assume that $D_i(\cdot)$ satisfies sufficient conditions to ensure a unique solution to sellers' pricing.

Let $V(p, n)$ denote a consumer's indirect utility from choosing its favourite product among n variants when each variant is priced at p (symmetric sellers set the same price). We assume that the indirect utility is increasing and concave in variety n, and decreasing in price p, respectively:

Assumption 1 *(i).* $\frac{\partial V(p,n)}{\partial p} < 0$, *(ii)* $\frac{\partial V(p,n)}{\partial n} > 0$, *and (iii)* $\frac{\partial^2 V(p,n)}{\partial n^2} < 0$.

These assumptions are the same as our Assumptions (A4') and (A5') in the multi-product monopoly analysis with search costs, see 3. Intuitively, when the platform offers more variants, consumers find a better match to their tastes on the platform and more variety gives extra benefits at a decreasing rate as better (average) matches generate decreasing returns.

4.1 Equilibrium Analysis

We now characterize the Subgame Perfect Nash equilibrium of the three-stage game by backward induction.

4.1.1 Seller and buyer participation

After participation decisions, that is, given $n \geq 2$ sellers and $F(\tilde{\tau}) > 0$ buyers are on the platform, each seller sets its price to maximize its variable profit as a reaction to the vector of rival sellers' prices $\mathbf{p_{-i}}$:

$$\max_{p_i} \pi_i = (p_i - c - w)D(p_i, \mathbf{p_{-i}})$$

By symmetry each seller sets the same price in equilibrium. We assume that this price is a well-defined function of n and w, $p^*(n, w)$, in the domain $n \geq 2$ and $w \in R$. When $n = 1$, there is a monopoly seller on the platform and it sets the monopoly price, which we denote by $p^*(1, w)$. Let $\pi^*(n, w)$ denote the per-seller per-buyer variable profit in equilibrium when there are n sellers on the platform and each sets price $p^*(n, w)$. We assume that as the number of sellers increases, the equilibrium price decreases:

Assumption 2 $\frac{\partial p^*}{\partial n} < 0$.

Intuitively, a bigger number of differentiated sellers (variants) implies more intense competition and so lower margins. This assumption holds for commonly used demand specifications of differentiated competition, for instance, Multinominal Logit demand, Vickrey-Salop circle demand.

We identify two equilibrium conditions that will determine the equilibrium number of sellers and buyers on the platform given the platform's fees, $\{\phi, w\}$. The first condition is the zero-profit condition for each seller (free-entry condition), which determines the number of sellers on the platform:

$$\pi^*(n, w)F(\tilde{\tau}) = \phi + K \quad . \tag{40}$$

The second condition determines the number of buyers on the platform. The marginal type $\tilde{\tau}$ is equal to the expected indirect utility from participating to the platform:

$$\tilde{\tau}(n^e, w) = V(n^e, p^*(n^e, w))]. \tag{41}$$

Observe that $\tilde{\tau}(n^e, w)$ is an increasing function of n^e due to Assumption 1 and Assumption 2: $\frac{d\tilde{\tau}}{dn^e} = \frac{\partial V}{\partial n^e} + \frac{\partial V}{\partial p^*}\frac{\partial p^*}{\partial n^e} > 0$ since $\frac{\partial V}{\partial n^e} > 0$, $\frac{\partial V}{\partial p^*} < 0$ by Assumption 1 and $\frac{\partial p^*}{\partial n^e} < 0$ by Assumption 2.

We assume that the platform makes positive profits if one seller is active on the platform than having no participation on both sides. This is the case if sellers' fixed cost, K, and marginal cost, c, are not too high:

Assumption 3 $\pi^*(1,0)F(\tilde{\tau}(1,0)) - K > 0$

where the amount of buyers on the platform is $\tilde{\tau}(1,0) = E[V(1,p^*(1,0))]$. The assumption implies that the continuation outcome with zero participation on both sides is pareto-dominated by the outcome with a monopoly seller on the platform. Hence, from now on we mainly focus on the case of $n \geq 1$.

We first prove the following:

Lemma 6 *If $F(\tau)$ is weakly concave and $\pi^*(n,w)$ is decreasing and log-concave in n then the zero-profit condition of sellers, (40), implies an increasing and (weakly) convex function of $\tilde{\tau}(n)$.*

Using the lemma we then characterize the continuation equilibrium participation by buyers and sellers given the platform's fees $\{w, \phi\}$:

Proposition 8 *If $F(\tau)$ is weakly concave and $\pi^*(n,w)$ is decreasing and log-concave in n then there exists at most three subgame equilibria to the sellers' and buyers' participation decisions given the platform's fees $\{w, \phi\}$ (due to symmetry the platform charges the same fee to all sellers). One with zero participation on both sides, the second equilibrium with a lower number of participants on both sides than the third equilibrium, where the second one is not stable.*

Figure 1 illustrates the continuation equilibrium participation levels of buyers and sellers. First, note that zero participation on both sides, E_1, cannot prevail in equilibrium of the entire game due to Assumption 3. In general there exists two intersections of the zero-profit condition of sellers (red curve) and the consumer participation condition (green curve), since the former is an increasing and weakly convex function of n (by Lemma 6), and the latter is an increasing function of n by Assumptions 2 and 2 as we illustrated above.[5] Note that the interior equilibrium with the lower number of sellers and buyers, E_2, is not stable, since starting from that equilibrium if we increase

[5] In the graph we draw the curve implied by the consumer participation constraint as a concave function of n, which does not have to be the case in general. What we need is that this curve does not coincide with the curve implied by the zero-profit condition, which would happen only in very special case and so we outlaw this.

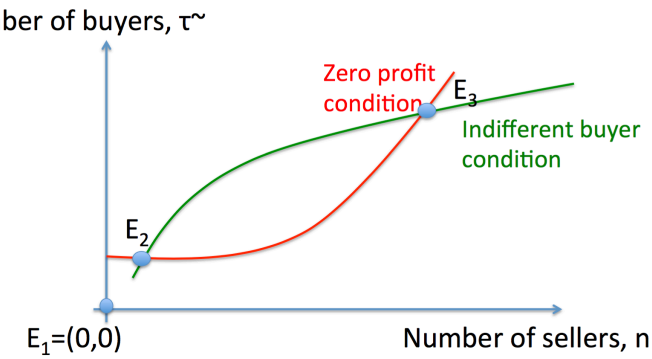

Figure 1: The equilibrium number of sellers and buyers

the number of sellers by ϵ, consumers will be better-off with a higher number of sellers (the marginal consumer type increases) and the sellers will also be better-off since the increased marginal type implies more consumers on the platform and so more expected profits. The new equilibrium will then be the interior equilibrium with the higher number of sellers and buyers, E_3.

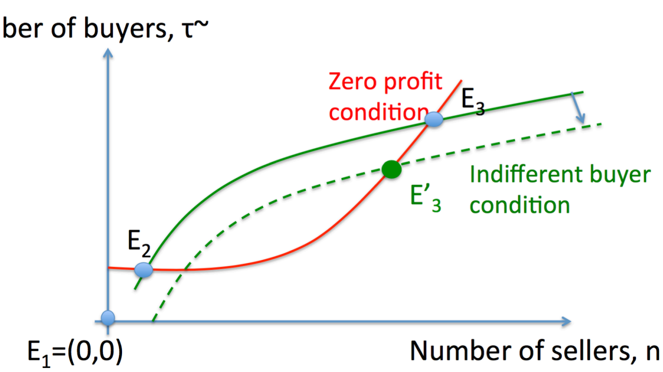

Figure 2: The effect of increasing w on the equilibrium number of buyers and sellers (ex-post covered market).

Figure 2 illustrates how an increase in the per-transaction fee, w, affects the equilibrium participation levels in an ex-post covered market. Sellers increase their price by the amount of the fee change (full cost passthrough). Thus, this will not affect the zero-profit curve. However, the increase in w raises the consumers' expected seller price and so will lower the expected consumer surplus of participating (the green curve shifts downwards). As a result, the number of buyers and the number of sellers on the platform decrease. The new stable equilibrium is at point E_3'. If the market is not covered, in standard demand models (e.g., log-concave demand) sellers increase their price less than the wholesale price increase (partial cost passthrough), and so sellers' margin will decrease, which in turn lower their variable profit and lead to the red curve to shift upwards. At the same time, the raised seller prices lower the expected consumer surplus of participating (the green curve shifts downwards). As Figure 3 illustrates the new equilibrium will be at E_3', which might have even fewer number of buyers and sellers than the ex-post covered market

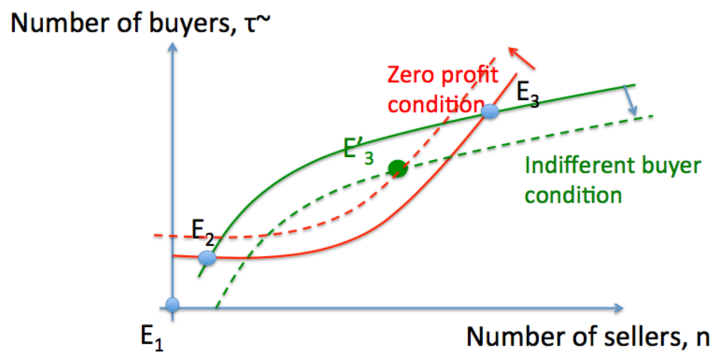

Figure 3: The effect of increasing w on the equilibrium number of buyers and sellers.

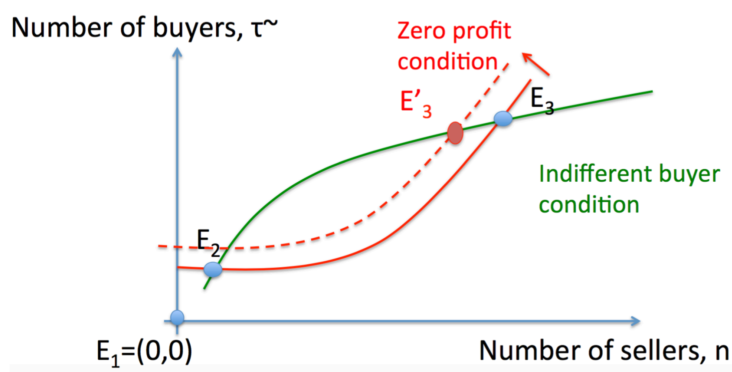

Figure 4: The effect of increasing ϕ on the equilibrium number of buyers and sellers.

Figure 4 illustrates how an increase in the listing fee, ϕ, affects the equilibrium participation levels. Fewer sellers enter the platform at a higher listing fee (the red curve shifts upwards). As a result, the number of buyers and sellers on the platform decrease. The new stable equilibrium is at point E_3'.

Note that the above results do not depend on the fact that sellers' transaction fee is constant per unit. The qualitative results would be the same if we allowed the platform to charge seller commissions proportional to seller revenue. The only difference of proportional commissions would be sellers' profit expression, $\pi_i = ((1 - w)p_i - c)D(p_i, \mathbf{p_{-i}})$, and the equilibrium price that is calculated by maximizing the latter profit.

4.1.2 The equivalence of the platform's problem to a multiproduct monopolist's

The platform sets (w, ϕ) to maximize its profit,

$$\Pi^{pl}(w, \phi) = nwD(p^*(n, w), n)F(\tilde{\tau}) + \phi n, \tag{42}$$

which is the sum of fees collected from sellers: transaction fees over the total volume of trade plus the fixed seller fees. The platform maximizes this profit subject to the equilibrium participation conditions of sellers (40) and buyers (41). Per-seller demand by each participant on the platform is $D(p^*(n, w), n)$, where $p^*(n, w)$ is the symmetric price set by each seller when there are n sellers on the platform.

Consider now the multiproduct monopolist model of Section 3, where consumers incur search costs to learn their tastes for products and the firm sells n symmetrically differentiated products to mass 1 of consumers. Recall that consumers observe p and n before visiting the store. Consumer cost τ is distributed over a compact interval $[0, \bar{\tau}]$ with cdf $F(.)$ and pdf $f(.)$. Assuming the cost of each variety is K, the monopolist's profit is

$$\Pi(p, n) = \pi(p, n)F(\tilde{\tau}) - nK, \tag{43}$$

where $\pi(p, n)$ denotes per-product per-customer profit, $\pi(p, n) = n(p - c)D(p, n)$, when there are n symmetrically differentiated products at price p and each customer's demand for each product is $D(p, n)$. The marginal consumer visiting the store is $\tilde{\tau}$, which is equal to the indirect utility of choosing one product from n variants that are priced at p:

$$\tilde{\tau} = V(p, n). \tag{44}$$

Recall that we assume (A4') and (A5') for $V(p, n)$: $\frac{\partial V(p,n)}{\partial n} > 0$, $\frac{\partial^2 V(p,n)}{\partial n^2} < 0$ and $\frac{\partial V(p,n)}{\partial p} < 0$. The monopolist's optimal variety and price will then be the solution to the following equilibrium conditions:

$$\frac{d\Pi}{dp} = \partial_p \pi F(\tilde{\tau}) + \pi(n,p) f(\tilde{\tau}) \partial_p V = 0, \tag{45}$$

$$\frac{d\Pi}{dn} = \partial_n \pi F(\tilde{\tau}) + \pi(n,p) f(\tilde{\tau}) \partial_n V - K = 0 \tag{46}$$

We assume the second-order conditions of the monopolist's problem are satisfied:

Assumption 4 $\frac{d^2\Pi}{dp^2} < 0$, $\frac{d^2\Pi}{dn^2} < 0$, *and* $\frac{d^2\Pi}{dp^2}\frac{d^2\Pi}{dn^2} - (\frac{d^2\Pi}{dpdn})^2 > 0$.

The first condition, (45), then implies the monopolist equilibrium price as a function of its variety: $p^*(n)$. We assume that the monopolist's equilibrium price is increasing in the number of variants it offers: $\frac{dp^*}{dn} > 0$. This will be the case under Assumption 4 and the following assumption:[6]

Assumption 5 $\frac{d^2\Pi}{dpdn} > 0$.

These assumptions hold for commonly used demands of differentiated products, such as Multinominal Logit and Vickrey-Salop circle models (as we illustrate below in the examples). Intutively, the monopolist's optimal price for each variety increases in the number of variants it offers since the monopolist can serve consumers' taste better when it offers more variants and so could capture increased willingness-to-pay for its products by raising its price for each variant.

We are now ready to present the equivalence between the platform's problem and the multiproduct monopolist's problem:

Proposition 9 *Suppose $\pi^*(n, \mathbf{w})F(V(n, p^*(n, \mathbf{w})))$ is a real-valued continuous and invertible function of n to $\mathbf{R}^+$ (at a given $\mathbf{w} \in \mathbf{R}$). The platform's problem of choosing a per-transaction seller fee w and a fixed seller fee ϕ to maximize (42) subject to the equilibrium participation conditions of sellers (40) and buyers (41) is equivalent to the problem of the multiproduct monopolist choosing p and n to maximize its profit, (43), subject to consumers' participation condition, (44).*

We prove the result in two steps. First, we show that capturing sellers' surplus via a fixed fee enables the platform to internalize the entire profits generated from trade

[6] By taking the total derivative of condition (45) we obtain $\frac{dp^*}{dn} = -\frac{\frac{d^2\Pi}{dpdn}}{\frac{d^2\Pi}{dp^2}} > 0$ since the denominator is negative by Assumption 4 and the numerator is positive by Assumption 5.

on the platform and therefore the platform's profit expression corresponds to the one of the multiproduct monopolist selling n symmetrically differentiated products. Second, we show how different observability assumptions of these problems induce the same outcome. More precisely, in the platform model we assume that consumers observe sellers' fees set by the platform before choosing whether to enter the platform. They observe the number of products, products' prices, and their match value to each product only after they enter the platform. On the other hand, in the multiproduct monopolist problem we assume that consumers observe the number of products and their prices before visiting the store, and realize their match value to each product only once they are in the store. In the second part we prove that even if consumers do not observe the number of products and their prices before entering the platform, they could infer them perfectly from observing the platform's seller fees. Each pair of seller fees corresponds to a unique price and a unique variety if the per-seller profit is continuous and invertible in variety. This result implies that the platform's optimal seller fees induce the number of products (variety) and the marginal consumer type that would be chosen by the multiproduct monopolist. In other words, the platform is able to coordinate sellers' pricing by using its seller fees.

Note that the equivalence result is robust to allowing the platform to charge proportional seller commissions. To see this consider the platform's profit if it sets a seller fee proportional to sellers' revenue as well as a fixed seller fee:

$$\Pi^{pl}(w, \phi) = nwp^*(n, w)D(p^*(n, w), n)F(\tilde{\tau}) + \phi n. \tag{47}$$

By replacing the equality of ϕ from the sellers' participation constraint:

$$\phi = ((1 - w)p^*(n, w) - c)D(p^*(n, w), n)F(\tilde{\tau}) - K, \tag{48}$$

we can rewrite the platform's profit as:

$$\Pi^{pl}(n, w) = n(p^*(n, w) - c)D(p^*(n, w), n)F(\tilde{\tau}) - nK, \tag{49}$$

and thus prove the equivalence result as we did above.

Corollary 3 *The equivalence between the platform's problem and the multiproduct monopolist's problem holds if the platform charges each seller a commission proportional to sales revenue and a fixed fee.*

The equivalence result has important implications for platforms which intermediate trade between buyers and sellers. The result shows that in equilibrium the platform eliminates competition between sellers by raising sellers' transaction fees sufficiently high, since the platform can capture sellers' profits via fixed seller fees. The equivalence result enables us to apply all the results that we derived on variety provision by multiproduct monopolist, the results of Sections 2 and 3, in the context of a monopoly trade platform.

We will illustrate the equivalence result with different demand specifications below: Ex-post covered market demand and MNL demand. In the MNL analysis we will also illustrate that the equivalence results hold also for asymmetrically differentiated sellers. In that case we also derive results on how the platform's seller fees differ across sellers with different qualities and whether this might cause any distortion by limiting access of high quality sellers to the platform.

4.2 Examples: Ex-post covered market

We characterize the platform's optimal fees (w, ϕ) that implement the multiproduct monopolist's price and variety choice in the example of an ex-post covered market, that is, when all consumers who are on the platform purchase a product. In the multiproduct monopolist model, ex-post covered market means that all consumers who visit the shop purchase a product. Ex-ante the market is not covered, that is, some fraction of consumers does not enter the platform or visit the shop.

In an ex-post covered market the indirect utility is additively separable in price and benefit from variety: $V(p,n) = B(n) - p$, where $B(n)$ denotes the consumer benefit of optimal consumption when there are n symmetric variants (the number of differentiated sellers) to choose from. We expect quite generally that $B(n)$ is increasing and concave: more variety gives extra benefits at a decreasing rate as better (average) matches generate decreasing returns. Using the marginal type definition, $\tilde{\tau} = B(n) - p$, we express the price of each variant in terms of the marginal type: $p = B(n) - \tilde{\tau}$. We replace the equality of price into the multiproduct monopolist's profit and rewrite it as a function of variety and the marginal consumer type:

$$\Pi(n, \tilde{\tau}) = (B(n) - \tilde{\tau} - c)F(\tilde{\tau}) - nK, \tag{50}$$

We then re-express the multiproduct monopolist's problem as maximizing (50) subject to n and $\tilde{\tau}$. The first-order conditions with respect to n and $\tilde{\tau}$ are, respectively,

$$B'(n)F(\tilde{\tau}) = K, \tag{51}$$

$$(B(n) - c - \tilde{\tau})f(\tilde{\tau}) = F(\tilde{\tau}), \tag{52}$$

which determine the monopolist's optimal choice for variety, n^*, and the marginal consumer type, $\tilde{\tau}^*$. The optimal variety equates the marginal benefit of variety, $B'(n)F(\tilde{\tau})$, to its cost, K. The optimal utility to the marginal consumer equates the gains from increasing the utility of the marginal type, that is, the gains from consumers at the extensive margin, $(B(n)-c-\tilde{\tau})f(\tilde{\tau})$, to the cost of offering a higher utility to the marginal type, that is, the unit margin loss from existing customers: $F(\tilde{\tau})$.

In order to induce the multiproduct monopolist's choices, n^*, $\tilde{\tau}^*$, the platform needs to set w^* such that consumers' participation constraint, $\tilde{\tau} = B(n^e) - p^*(n^e, w)$, is equivalent to (52) in equilibrium where consumers have correct expectations about the number of sellers, $n^e = n^*$:

$$p^*(n^*, w^*) = c + \frac{F(\tilde{\tau}^*)}{f(\tilde{\tau}^*)}, \tag{53}$$

and the platform needs to set ϕ^* such that the sellers' participation constraint, $\frac{p^*(n,w)-c-w}{n}F(\tilde{\tau}) - K - \phi^* = 0$, is equivalent to (51):

$$\phi^* = \frac{(p^*(n^*, w^*) - c - w^*)K}{n^* B'(n^*)} - K. \tag{54}$$

Hence, we show that to implement the multiproduct monopolist's choice of n^* and $\tilde{\tau}^*$, the platform sets w^* and ϕ^*, which are given by (53) and (54), respectively. See Appendix E for the illustration of this result for Vickrey [1964] - Salop [1979] model of the competition between differentiated products. In this example we show that the platform's optimal transaction fee for sellers is increasing in consumers' reservation price, the number of sellers (variants), and the level of substitution between sellers' products. Intuitively, when there are many sellers or sellers' products are very close substitutes, sellers' margins would be very low in equilibrium. To counter balance too low seller prices, the platform raises its seller commission. By doing so the platform raises all sellers' unit cost and induces the prices that would be set by a multi-product monopolist, despite the fact that sellers are actively and independently compete on the platform.

4.3 Examples: Asymmetric MNL

We now illustrate that the equivalence result also holds with asymmetrically differentiated sellers in the MNL. Suppose the utility of buying seller i's product is

$$u_i = v_i - p_i + \mu\epsilon_i \tag{55}$$

where v_i denotes consumption utility from product i, p_i denotes product i's price, ϵ_i is the random taste shock, and μ measures differentiation between products. and the utility of not buying any product is $u_0 = \epsilon_0$.

We assume that random taste shocks (ϵ_i s and ϵ_0) are double exponentially distributed. The demand for seller i's product is then given by asymmetric Multinominal Logit (MNL):

$$\mathbb{P}_i = \frac{exp\left(\frac{v_i - p_i}{\mu}\right)}{\sum_{j=1}^{n} exp\left(\frac{v_j - p_j}{\mu}\right) + 1} \tag{56}$$

Consider the subgame where seller i accepted the platform's contract (w_i, ϕ_i). The problem of seller i is to set p_i taking the platform's fees as given:

$$\max_{p_i} \Pi_i = (p_i - c - w_i)\mathbb{P}_i - K - \phi_i \tag{57}$$

The first-order condition of this problem is

$$\frac{d\Pi_i}{dp_i} = \mathbb{P}_i + (p_i - c - w_i)\frac{d\mathbb{P}_i}{dp_i} = 0 \tag{58}$$

Using the properties of MNL we derive $\frac{d\mathbb{P}_i}{dp_i} = -\frac{\mathbb{P}_i(1-\mathbb{P}_i)}{\mu}$. Replacing this into the previous first-order condition gives seller i's optimal markup as a function of the platform's fees and other sellers' prices:

$$p_i^* - c = w_i + \frac{\mu}{1 - \mathbb{P}_i}. \tag{59}$$

Solving the problem of all sellers gives us n equations for n unknowns, for $i = 1, ..n$,

$$p_i^* - c = w_i + \frac{\mu}{1 - \frac{exp\left(\frac{v_i - p_i^*}{\mu}\right)}{\sum_{j=1}^{n} exp\left(\frac{v_j - p_j^*}{\mu}\right)+1}}.$$

The solution to the latter equations determines sellers' equilibrium prices as implicit functions of the platform's unit fees (w_i).

Anticipating sellers' pricing behavior, the platform sets (w_i, ϕ_i) to maximize its profit subject to sellers' participation constraint:

$$max_{w_i,\phi_i} \Pi = \sum_{i=1}^{n} (w_i \mathbb{P}_i^* + \phi_i)$$
$$\text{s.t. } (p_i^* - c - w_i)\mathbb{P}_i^* - K - \phi_i \geq 0 \text{ for all i.}$$

The participation constraints should be binding in equilibrium since otherwise the platform would increase its profit by raising ϕ_i. Replacing the binding constraints illustrates that the platform's problem is equivalent to setting w_i to maximize the total industry profit.

$$\max_{w_i} \Pi = \sum_{i=1}^{n} (p_i^* - c)\mathbb{P}_i^* - K \tag{60}$$

In other words, the platform's objective function corresponds to the multi-product monopolist's objective, since the platform can capture sellers' total profits via fixed fees. Given that the platform can control each seller's price via its unit fee, it can implement the optimal price choice of the multi-product monopolist, that is,

$$\max_{\{p_1,\ldots,p_n\}} = \Pi = \sum_{i=1}^{n} (p_i - c)\mathbb{P}_i - K \tag{61}$$

The first-order conditions of this problem are, for $i = 1, .., n$,

$$\frac{d\Pi}{dp_i} = \mathbb{P}_i + (p_i - c)\frac{d\mathbb{P}_i}{dp_i} + \sum_{j \neq i} (p_j - c)\frac{d\mathbb{P}_j}{dp_i} = 0 \tag{62}$$

Using the properties of the MNL we derive

$$\begin{aligned}\frac{d\mathbb{P}_i}{dp_i} &= -\frac{\mathbb{P}_i(1-\mathbb{P}_i)}{\mu} \\ \frac{d\mathbb{P}_j}{dp_i} &= \frac{\mathbb{P}_j\mathbb{P}_i}{\mu}.\end{aligned}$$

Replacing these into the first-oder conditions gives us the optimal markup for product i:

$$p_i - c = \mu + \sum_{j=1}^{n}(p_j - c)\mathbb{P}_j.$$

Thus, the multi-product monopolist wants to set the same markup for all products, for $i = 1, .., n$,

$$p_i^m - c = \frac{\mu}{1 - \sum_{j=1}^{n}\mathbb{P}_j}. \tag{63}$$

Given sellers' price reactions to platform fees (59), the platform sets w_i in order to implement the multi-product monopolist's optimal prices:

$$w_i^m = \mu\left(\frac{1}{1-\sum_{j=1}^{n}\mathbb{P}_j} - \frac{1}{1-\mathbb{P}_i}\right) \tag{64}$$

We have a couple of observations on the platform's optimal seller fees and resulting product prices. First, in equilibrium products' prices (markups) are the same and higher quality products have higher demand. Second, the platform charges lower unit fees on higher quality products than lower quality products given that $\mathbb{P}_i$ is higher for higher quality. Asymmetric sellers in equilibrium set different markups: higher quality sellers set higher markups. On the other hand, the platform wants to implement the same markup for all products. In order to achieve this, the platform sets lower unit commission on higher quality products. Third, the equilibrium fixed fee is higher for higher quality products:

$$\phi_i = \frac{\mu}{1-\sum_{j=1}^{n}\mathbb{P}_j}\mathbb{P}_i - K \tag{65}$$

We summarize these findings in the following:

Proposition 10 *Consider a platform facilitating interactions between buyers and asymmetrically differentiated sellers. When the demand for each product is given* *by the asymmetric MNL, the platform behaves like a multi-product monopolist by controlling prices via its seller commission and by controlling the number of products via its listing fees. The platform sets a lower unit commission and a higher fixed fee to a higher quality product seller than a lower quality product seller.*

We next consider selection of products into the platform. To do that we investigate how the platform's equilibrium seller fees for one product change if the platform replaces this product with a higher quality alternative. We study this by deriving the platform's optimal unit commission with respect to the demand of the product that the platform consider's replacing, say, $\mathbb{P}_i$:

$$\frac{dw_i^m}{d\mathbb{P}_i} = \mu \left(\frac{1}{\left(1 - \sum_{j=1}^{n} \mathbb{P}_j\right)^2} - \frac{1}{\left(1 - \mathbb{P}_i\right)^2} \right) > 0. \tag{66}$$

The latter inequality holds because the total quantity of sales is greater than the demand for product i, $\sum_{j=1}^{n} \mathbb{P}_j > \mathbb{P}_i$. This implies that the platform's optimal unit seller fee increases if it replaces one product with a higher quality alternative. Besides, observe that the platform's optimal fixed fee for product i also increases if it replaces this product with a higher quality (see equation(65)):

$$\frac{d\phi_i^m}{d\mathbb{P}_i} = \mu \frac{1}{\left(1 - \mathbb{P}_i\right)^2} > 0. \tag{67}$$

These observations imply that the platform sets higher seller fees to an entrant seller if the entrant wants to replace a lower quality product listed on the platform. This therefore would generate inefficient entry costs for higher quality sellers who would like to list their product on the trade platform. This ineffiency arises because when the platform sells a higher quality product, its total demand increases more than the demand for the replaced product. This in turn increases the discrepancy between the optimal monopoly markup that the platform would like to implement (63) and the markup chosen by individual sellers (59), and so calls for a higher unit commission to implement the monopoly markup. In other words, inefficiently high commissions to a better quality entrant seller is due to the platform behaving like a multi-product monopolist and setting the same markup for all products.

4.3.1 Percentage commissions and fixed fees

Suppose that the platform sets a percentage commission, or royalty r_i, and fixed fee, ϕ_i, to seller i. The problem of seller i is now to set p_i taking the platform's fees as given:

$$\max_{p_i} \Pi_i = (p_i(1 - r_i) - c)\mathbb{P}_i - K - \phi_i \tag{68}$$

The first-order condition of this problem is

$$\frac{d\Pi_i}{dp_i} = (1 - r_i)\mathbb{P}_i + (p_i(1 - r_i) - c)\frac{d\mathbb{P}_i}{dp_i} = 0 \tag{69}$$

Using the properties of MNL we derive $\frac{d\mathbb{P}_i}{dp_i} = -\frac{\mathbb{P}_i(1-\mathbb{P}_i)}{\mu}$. Replacing this into the previous first-order condition gives seller i's optimal markup as a function of the platform's fees and other sellers' prices:

$$p_i^* - c = c\frac{r_i}{1 - r_i} + \frac{\mu}{1 - \mathbb{P}_i}. \tag{70}$$

Solving the problem of all sellers gives us n equations for n unknowns, for $i = 1, ..n$,

$$p_i^* - c = c\frac{r_i}{1 - r_i} + \frac{\mu}{1 - \frac{exp\left(\frac{v_i - p_i^*}{\mu}\right)}{\sum_{j=1}^{n} exp\left(\frac{v_j - p_j^*}{\mu}\right) + 1}}.$$

The solution to the latter equations determines sellers' equilibrium prices as implicit functions of the platform's royalties (r_i).

Anticipating sellers' pricing behavior, the platform sets (r_i, ϕ_i) to maximize its profit subject to sellers' participation constraint:

$$max_{r_i,\phi_i} \Pi = \sum_{i=1}^{n} (p_i^* r_i \mathbb{P}_i^* + \phi_i)$$
$$\text{s.t. } (p_i^*(1 - r_i) - c)\mathbb{P}_i^* - K - \phi_i \geq 0 \text{ for all i.}$$

The participation constraints should be binding in equilibrium since otherwise the platform would increase its profit by raising the fixed fee of any non-binding constraint. Replacing the binding constraints illustrates that the platform's problem is equivalent to setting r_i to maximize the total industry profit.

$$\max_{r_i} \Pi = \sum_{i=1}^{n} (p_i^* - c)\mathbb{P}_i^* - K \tag{71}$$

In other words, the platform's objective function corresponds to the multi-product monopolist's objective, since the platform can capture sellers' total profits via fixed fees. Given that the platform can control each seller's price via its commission, it can implement the optimal price choice of the multi-product monopolist given in (63):

$$r_i^m = \frac{\mu\left(\frac{1}{1-\sum_{j=1}^{n}\mathbb{P}_j} - \frac{1}{1-\mathbb{P}_i}\right)}{c + \mu\left(\frac{1}{1-\sum_{j=1}^{n}\mathbb{P}_j} - \frac{1}{1-\mathbb{P}_i}\right)}. \tag{72}$$

Define $X \equiv \mu\left(\frac{1}{1-\sum_{j=1}^{n}\mathbb{P}_j} - \frac{1}{1-\mathbb{P}_i}\right)$, then $r_i^m = \frac{X}{c+X}$. Observe that X corresponds to the platform's optimal unit fee from the previous analysis ($w_i^m = X$ in the case of two-part tariffs with constant unit fees) and that the platform's optimal seller royalty r_i^m increases in X. These together imply that we have similar comparative statics on equilibrium prices: (1) products' prices (markups) are the same and higher quality products have higher demand, (2) the platform charges lower commissions on higher quality products than lower quality products, (3) the equilibrium fixed fee is higher for higher quality products, (4) the platform's optimal commission and fixed seller fees increase if it replaces one product with a higher quality alternative.

4.3.2 Unobserved seller heterogeneity

Now suppose that the platform cannot observe quality differentials between sellers and sets one two-part tariff w, ϕ to all sellers.

Appendices

A Vickrey-Salop Model

We use Vickrey [1964] - Salop [1979] as a model of the competition between n symmetrically differentiated products to illustrate an example where the monopolist over-provides variety compared to the second-best optimal variety (and also with respect to the first-best).

Mass 1 of consumers are uniformly located on the unit circle. In this model suppose consumers' reservation price is R, unit transportation cost is t and the market is covered. A consumer who is located x units away from one variant needs to pay transportation cost (distaste cost) tx if she travels to that variant (that is, if she consumes that variant). Consumers' benefit from choosing their preferred product among n variants is then $B(n) = R - \frac{t}{4n}$, where t is the parameter measuring differentiation between the products. Assume for the moment that consumers know their tastes without incurring search costs.

The first-best optimal variety is the one that equates the marginal benefit of variety to the marginal cost of variety:

$$\frac{t}{4(n^{FB})^2} = K,$$

so $n^{FB} = \sqrt{\frac{t}{4K}}$. Price is a fixed transfer between consumers and the firm, so the level of price does not affect the total welfare (due to the market-coverage assumption).

In equilibrium the monopolist will set the highest price that keeps consumers located in the mid-point between two variants indifferent between buying either product or not purchasing:

$$p^*(n) = R - \frac{t}{2n}$$

and chooses the variety that maximises her profit:

$$\Pi(n) = p^*(n) - c - nK = R - \frac{t}{2n} - c - nK,$$

which is basically setting n^* that satisfies the first-order condition:

$$\frac{t}{2(n^*)^2} = K,$$

so $n^* = \sqrt{\frac{t}{2K}}$, which is higher than the first-best optimal variety, n^{FB}: the monopolist over-provides variety compared to the first-best level.

Consumer surplus at the price chosen by the monopolist is

$$CS(n) = B(n) - p^*(n) = \frac{t}{2n} - \frac{t}{4n} = \frac{t}{4n}.$$

Observe that consumer surplus is decreasing in variety at the price chosen by the monopolist. Thus, the monopolist over-provides variety compared to the second-best optimal level.

B Multinominal logit model:

If we model the demand for a seller using Multinominal Logit demand, each buyer gets utility u_i from purchasing seller i's product, $i = 1, 2, .., n$:

$$u_i = v - p_i + \mu\epsilon_i, \tag{73}$$

where v denotes the unit consumption value, p_i denotes the price of seller i, ϵ_i is the taste parameter which is assumed to be i.i. double exponentially distributed across sellers, and seller differentiation is measured by parameter μ , which is assumed to be positive. We allow for (exogenous) outside option for buyers by assuming that a buyer gets u_0 if she does not buy from any seller on the platform: $u_0 = v_0 + \epsilon_0$, where v_0 denotes the value of the outside good, the taste for the outside good, ϵ_0, is assumed to be i.i. double exponentially distributed along with the ϵ_i. Under these assumptions the purchase probability for product i is given by [Anderson et al., 1992]:

$$\mathbb{P}_i \equiv \mathbb{P}(p_i, p_{-i}, n) = \frac{\exp((v - p_i)/\mu)}{\sum_{j=1}^{n} \exp((v - p_j)/\mu) + \exp(v_0/\mu)}. \tag{74}$$

Seller i's demand per consumer on the platform is therefore equal to $\mathbb{P}_i$. The marginal buyer $\tilde{\tau}$ is equal to the expected consumption utility:[7]

$$\tilde{\tau} = \mu ln \left(\sum_{j=1}^{n} \exp(\frac{v - p_j}{\mu}) + \exp(\frac{v_0}{\mu}) \right). \tag{75}$$

C Proof of Lemma 6

The total derivation of condition (40) gives us

$$\frac{d\tilde{\tau}}{dn} = -\frac{\frac{\phi+K}{\pi^2}\frac{d\pi}{dn}}{F'(\tilde{\tau})} > 0, \tag{76}$$

since $\frac{d\pi}{dn} < 0$ by assumption and $F'(.) > 0$. By taking the derivative of the latter with respect to n we obtain:

$$\begin{aligned}
\frac{d^2\tilde{\tau}}{dn^2} &= \frac{\left[\frac{2(\phi+K)}{\pi^3}\left(\frac{d\pi}{dn}\right)^2 - \frac{\phi+K}{\pi^2}\frac{d^2\pi}{dn^2}\right]F'(\tilde{\tau}) + \frac{\phi+K}{\pi^2}\frac{d\pi}{dn}F''(\tilde{\tau})\frac{d\tilde{\tau}}{dn}}{(F'(\tilde{\tau}))^2} \\
&= \frac{\phi+K}{\pi^2}\frac{\left[2\left(\frac{d\pi}{dn}\right)^2 - \pi\frac{d^2\pi}{dn^2}\right]\frac{F'(\tilde{\tau})}{\pi} + \frac{d\pi}{dn}F''(\tilde{\tau})\frac{d\tilde{\tau}}{dn}}{(F'(\tilde{\tau}))^2} \\
&= \frac{\phi+K}{(F'(\tilde{\tau}))^3\pi^3}\left(\left[2\left(\frac{d\pi}{dn}\right)^2 - \pi\frac{d^2\pi}{dn^2}\right]\left(F'(\tilde{\tau})\right)^2 - \left(\frac{d\pi}{dn}\right)^2 F''(\tilde{\tau})\frac{\phi+K}{\pi}\right)
\end{aligned}$$

where the last equality is obtained after replacing the equality of (76) and re-arranging the terms. When $F(\cdot)$ is concave, we have $F''(\tilde{\tau}) < 0$. When $\pi(n)$ is log-concave, we have $2\left(\frac{d\pi}{dn}\right)^2 - \pi\frac{d^2\pi}{dn^2} > 0$. Given that $F'(\tilde{\tau}), \pi(n) > 0$, we prove that when $F(\cdot)$ is concave and $\pi(n)$ is decreasing and log-concave, we have $\frac{d^2\tilde{\tau}}{dn^2} > 0$.

D Proof of Proposition 9

The platform maximizes its profit (42) subject to the equilibrium participation conditions of sellers (40) and buyers (41). From the participation condition of sellers, we

[7]See Anderson et al. [1992], p. 231, for the derivation of the expected utility from consumption in the logit oligopoly model with outside good.

have $\phi = \pi^*(n, w)F(\tilde{\tau}) - K$. Using this we rewrite the platform's profit as

$$\Pi^{pl} = \left[nwD(p^*(n, w), n) + n\pi^*(n)\right]F(\tilde{\tau}) - nK$$

Since sellers are symmetric, n times per-seller per-buyer profit is equal to the total seller markup per buyer: $\pi^*(n) = n(p^*(n, w) - c - w)D(p^*(n, w), n)$. Using the latter equality, we re-express the platform's problem as

$$\max_{w,n} \Pi^{pl} = n(p^*(n, w) - c)D(p^*(n, w), n)F(\tilde{\tau}) - nK$$

subject to $\tilde{\tau} = E[V(n, p^*(n, w))]$. The transaction fee, w, affects the platform's profit in (D) only via changing the equilibrium seller price p^* and for a given n, $p^*(n, w)$ is induced by at least one w ($p^*(n, w)$ is a function of w).[8] Thus, when consumers observe w, for a given expected variety, n^e, they could anticipate the equilibrium price: $p^*(n^e, w)$. Moreover, given that consumers see the platform's fixed seller fee, ϕ, they could anticipate correctly the total number of sellers (variety) in equilibrium from $\phi = \pi^*(n^e, \mathbf{w})F(V(n^e, p^*(n^e, \mathbf{w}))) - K$ if $\pi^*(n, \mathbf{w})F(V(n, p^*(n, \mathbf{w})))$ is a continuous, one-to-one (injective) and onto (so invertible) function of n to $\mathbf{R}^+$. More precisely, when this function is onto, any $\phi \in \mathbf{R}^+$ corresponds to a value of $\pi^*(n, \mathbf{w})F(V(n, p^*(n, \mathbf{w})))$, and when the function is one-to-one, this value can be implemented by only one n, and so when consumers see ϕ and w, they could correctly associate a unique value of n and a unique value of p (assuming that they are fully rational, aware of the model's parameters and solve the model correctly). In other words, observability of ϕ and w by consumers is theoretically equivalent to observing n and p. Hence, we can write the platform's problem as choosing p and n to maximize its profit

$$\max_{p,n} \Pi^{pl} = n(p - c)D(p, n)F(\tilde{\tau}) - nK$$

subject to $\tilde{\tau} = V(p, n)$, which is equivalent to the multiproduct monopolist's problem.

[8]Note that we do not need to assume that $p^*(n, w)$ is an injection (one-to-one function) of w. Even if there are more than one w s inducing the same $p^*(n, w)$ (for a given n), this would imply multiple solutions for the platform's optimal w, but these solutions would induce the same seller price and so would lead to the same outcome.

E Equivalence Result in Vickrey-Salop Model

We use Vickrey [1964] - Salop [1979] as a model of the competition between differentiated products to illustrate the previous equivalence result. In this model suppose consumers' reservation price is R and unit transportation cost is t. Consumers' benefit from choosing their preferred product among n variants is then $B(n) = R - \frac{t}{4n}$, where t is the parameter measuring differentiation between the products. Assume also that τ is uniformly distributed over $[0, 1]$, so we have $F(\tau) = \tau$ and $f(\tau) = 1$.

First consider the multiproduct monopolist's problem. The monopolist's optimal variety, n^*, and marginal consumer type, $\tilde{\tau}^*$, are the solutions to its optimality conditions, given in (51) and (52), respectively

$$\frac{t}{4n^{*2}}\tilde{\tau}^* = K,$$
$$R - \frac{t}{4n^*} - c = 2\tilde{\tau}^*,$$

which have at most two solutions with positive number of sellers and buyers. We select the stationary solution, which has the highest number of participants on both sides. Note also that depending on the parameter values the market could be fully covered on the buyer side: $\tilde{\tau}^* \geq 1$, which is the case when consumers' reservation price is sufficently high compared to the differentiation between the products and sellers' marginal cost: $R - \frac{t}{4n} - c \geq 2$.

Next consider the platform's problem. Given the platform's fees, (w, ϕ), the seller price and per-buyer profit of each seller are respectively:

$$p^*(n, w) = c + w + \frac{t}{n}, \qquad \pi^*(n) = \frac{t}{n^2}, \tag{77}$$

which are both decreasing in the total number of sellers (variants, n). The sellers' zero-profit condition, (40), is then

$$\tilde{\tau} = \frac{(\phi + K)n^2}{t}.$$

The latter implies that the marginal type under which the zero-profit condition holds is an increasing and convex function of variety. Moreover, the consumers' participation condition, (41), is

$$\tilde{\tau} = E[R - \frac{5t}{4n} - c - w],$$

which implies that the expected indirect utility (the marginal type, $\tilde{\tau}$) is an increasing and concave function of variety (n). Thus, for the uniform distribution $F(\cdot)$, there are at most two subgame equilibrium solutions with positive number of participants on both sides. Assumption 3 holds if and only if the differentiation between the firms and consumers' reservation price are sufficiently high compared to the sellers' fixed cost and marginal cost:

$$\text{Assumption 3 (Vickrey-Salop): } t(R - \frac{5t}{4} - c) - K > 0. \tag{78}$$

Given the equilibrium seller price is $p^*(n, w) = c + w + \frac{t}{n}$, the platform's optimal w^* and ϕ^* induce the monopolist's optimal variety and marginal consumer type (as we showed above), and so satisfy the conditions, (53) and (54):

$$w^* = \frac{1}{2}(R - \frac{9t}{4n^*}) - c, \qquad \phi^* = 3K.$$

Note that the market is ex-post covered if $p^* \leq R - \frac{t}{2n}$. After replacing the equality for w^* into p^*, we can show that the market is ex-post covered if $R \geq \frac{3t}{4n^*}$, which we assume to be the case. The platform's optimal transaction fee for sellers is increasing in consumers' reservation price, the number of sellers (variants), and the level of substitution between sellers' products. Intuitively, when there are many sellers or sellers' products are very close substitutes, sellers' margins would be very low in equilibrium. To counter balance too low seller prices, the platform raises its seller commission. By doing so the platform raises all sellers' unit cost and induces the prices that would be set by a multi-product monopolist, despite the fact that sellers are actively and independently compete on the platform.

References

Simon P Anderson, Andre De Palma, and Jacques François Thisse. *Discrete choice theory of product differentiation*. MIT press, 1992.

Simon P Anderson, Andre De Palma, and Yurii Nesterov. Oligopolistic competition and the optimal provision of products. *Econometrica: Journal of the Econometric Society*, pages 1281–1301, 1995.

M. Bagnoli and T. Bergstrom. Log-concave probability and its applications, 1989. Mimeo, University of Michigan.

H Chamberlin Edward. The theory of monopolistic competition: A re-orientation of the theory of value, 1933.

Gregory S Crawford, Oleksandr Shcherbakov, and Matthew Shum. Quality overprovision in cable television markets. *forthcoming at American Economic Review*.

Avinash K Dixit and Joseph E Stiglitz. Monopolistic competition and optimum product diversity. *The American Economic Review*, 67(3):297–308, 1977.

Andrei Hagiu. Two-sided platforms: Product variety and pricing structures. *Journal of Economics & Management Strategy*, 18(4):1011–1043, 2009.

Kelvin Lancaster. *Variety, equity, and efficiency: product variety in an industrial society*, volume 10. New York: Columbia University Press, 1979.

N Gregory Mankiw and Michael D Whinston. Free entry and social inefficiency. *The RAND Journal of Economics*, pages 48–58, 1986.

Eric Maskin and John Riley. Monopoly with incomplete information. *The RAND Journal of Economics*, 15(2):171–196, 1984.

Michael Mussa and Sherwin Rosen. Monopoly and product quality. *Journal of Economic theory*, 18(2):301–317, 1978.

Volker Nocke, Martin Peitz, and Konrad Stahl. Platform ownership. *Journal of the European Economic Association*, 5(6):1130–1160, 2007.

Steven C Salop. Monopolistic competition with outside goods. *The Bell Journal of Economics*, pages 141–156, 1979.

A Michael Spence. Monopoly, quality, and regulation. *The Bell Journal of Economics*, pages 417–429, 1975.

William Vickrey. Principles of efficiency–discussion. In *American Economic Review–Papers and Proceedings*, volume 54, pages 88–96, 1964.